Three Phase Theory

Phil Fischbach

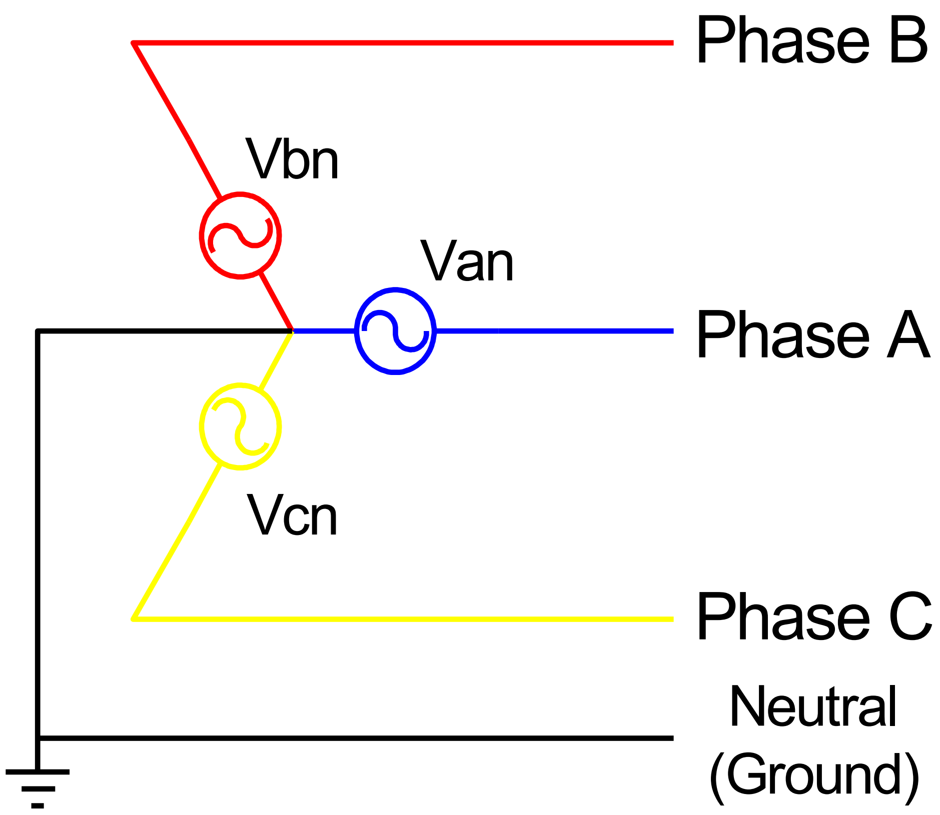
Mechanical Engineer

10737 Lexington Drive
Knoxville, TN 37932
Phone: (865) 966-5856

www.powermetrix.com

Three Phase Power

Introduction



Basic Assumptions

- Three AC voltage sources
- Voltages Displaced in time
- Each sinusoidal
- Identical in Amplitude

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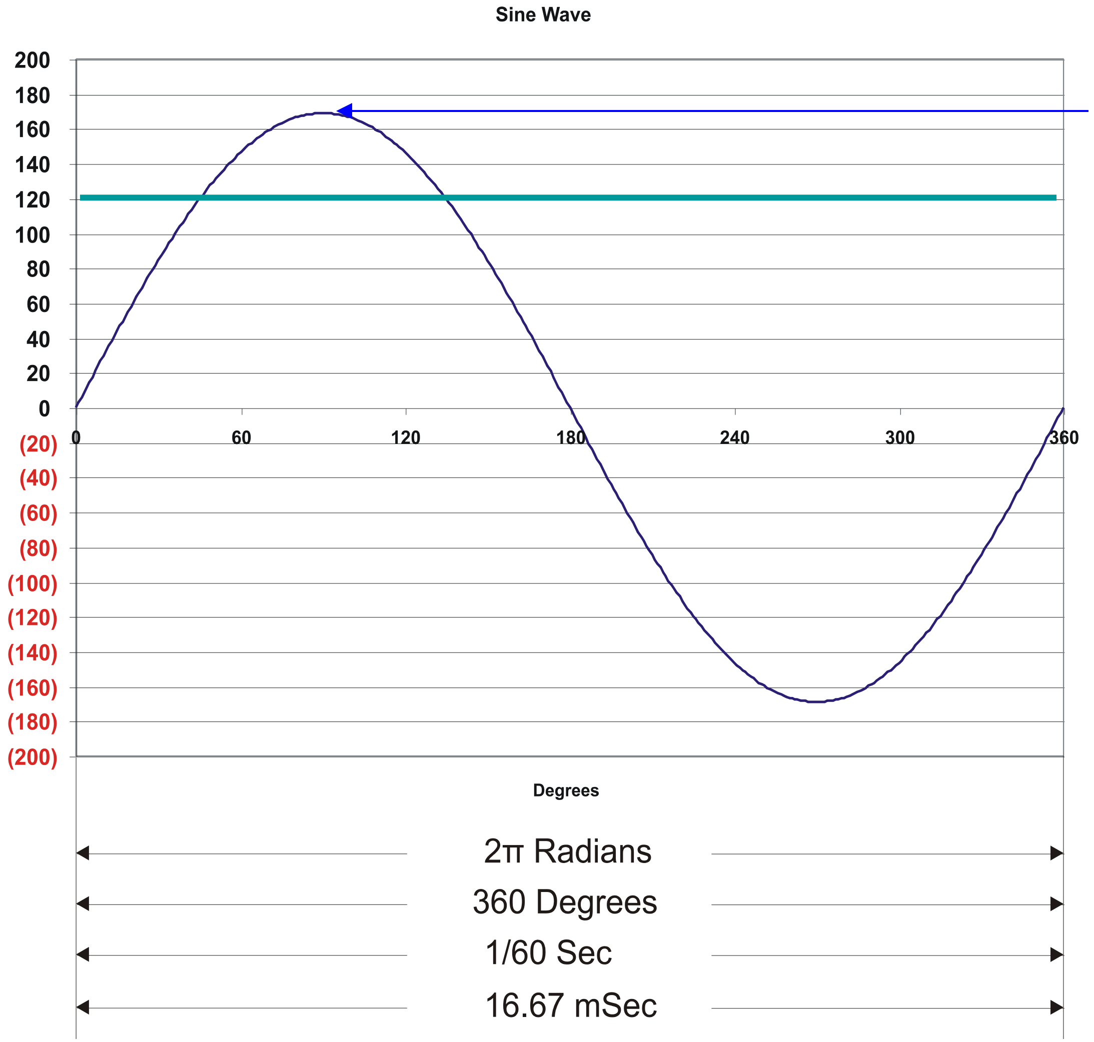
AC Theory – Sine Wave



$$V = V_{pk} Sin(2\pi ft - \theta)$$

$$V_{pk} = \sqrt{2} V_{rms}$$

$$V_{rms} = 120$$

$$V_{pk} = 169$$

$$f = 1 / time$$

$$\theta = 0$$

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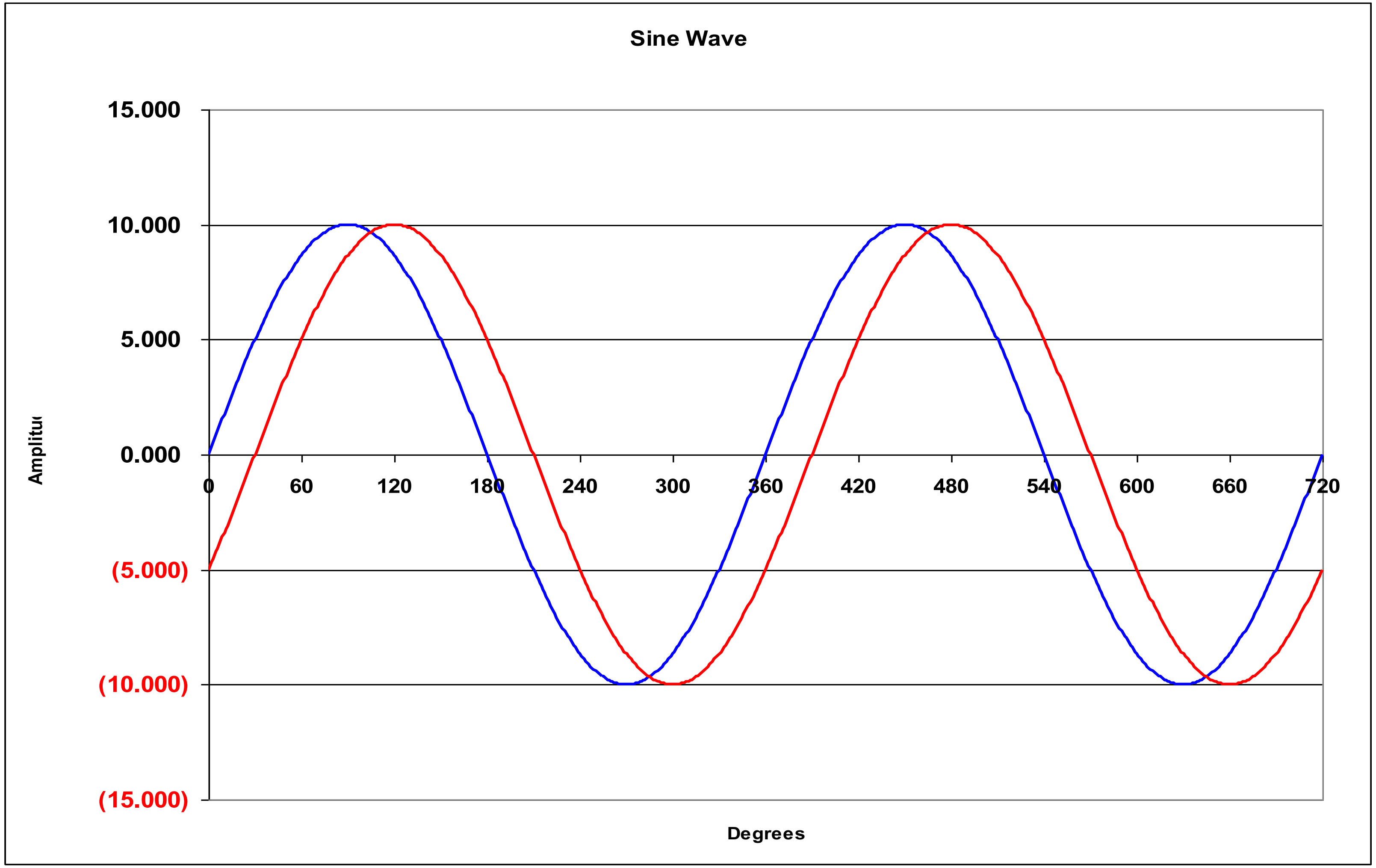
AC Theory - Phase



$$V = 10 Sin(2\pi ft)$$

$$V = 10 Sin(2\pi ft - 30)$$

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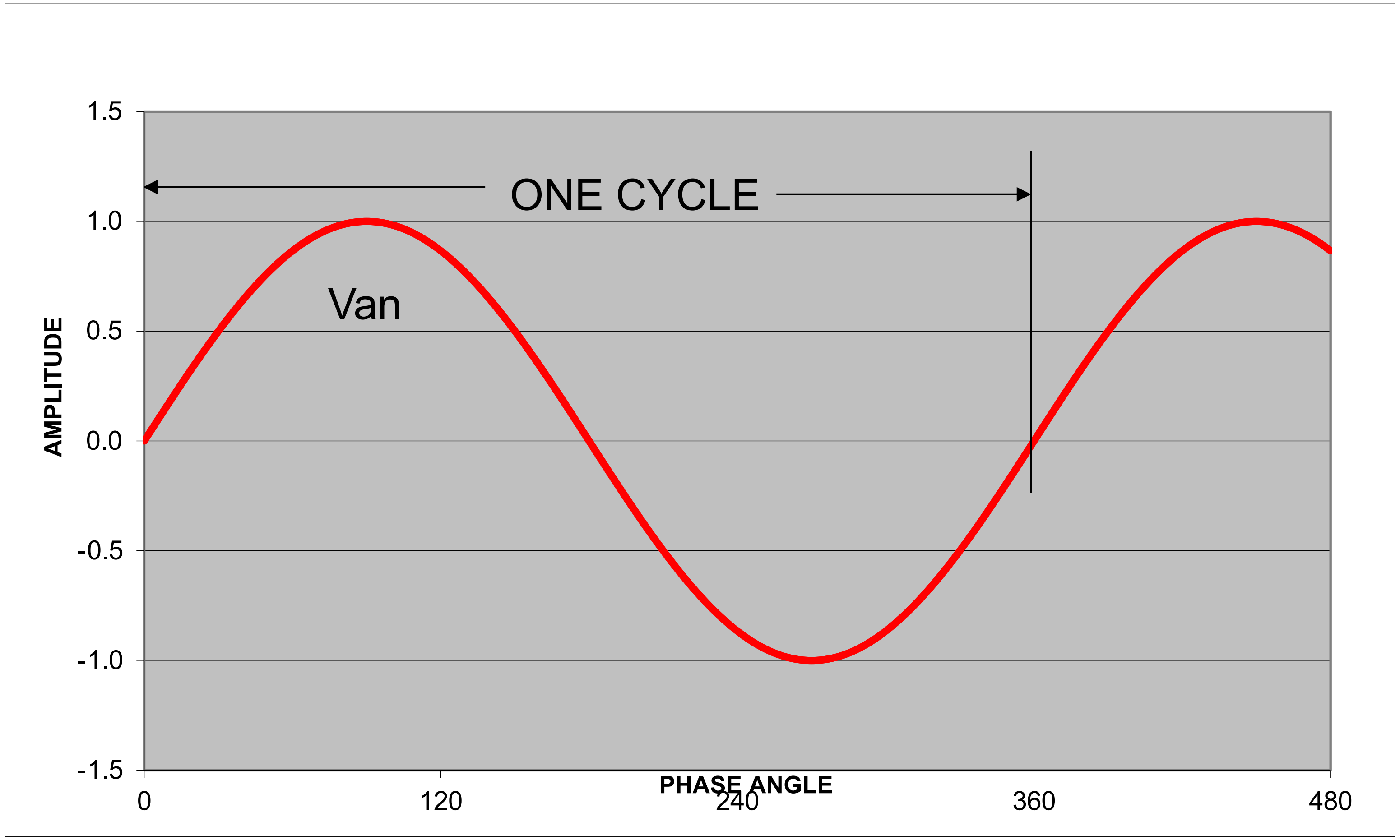
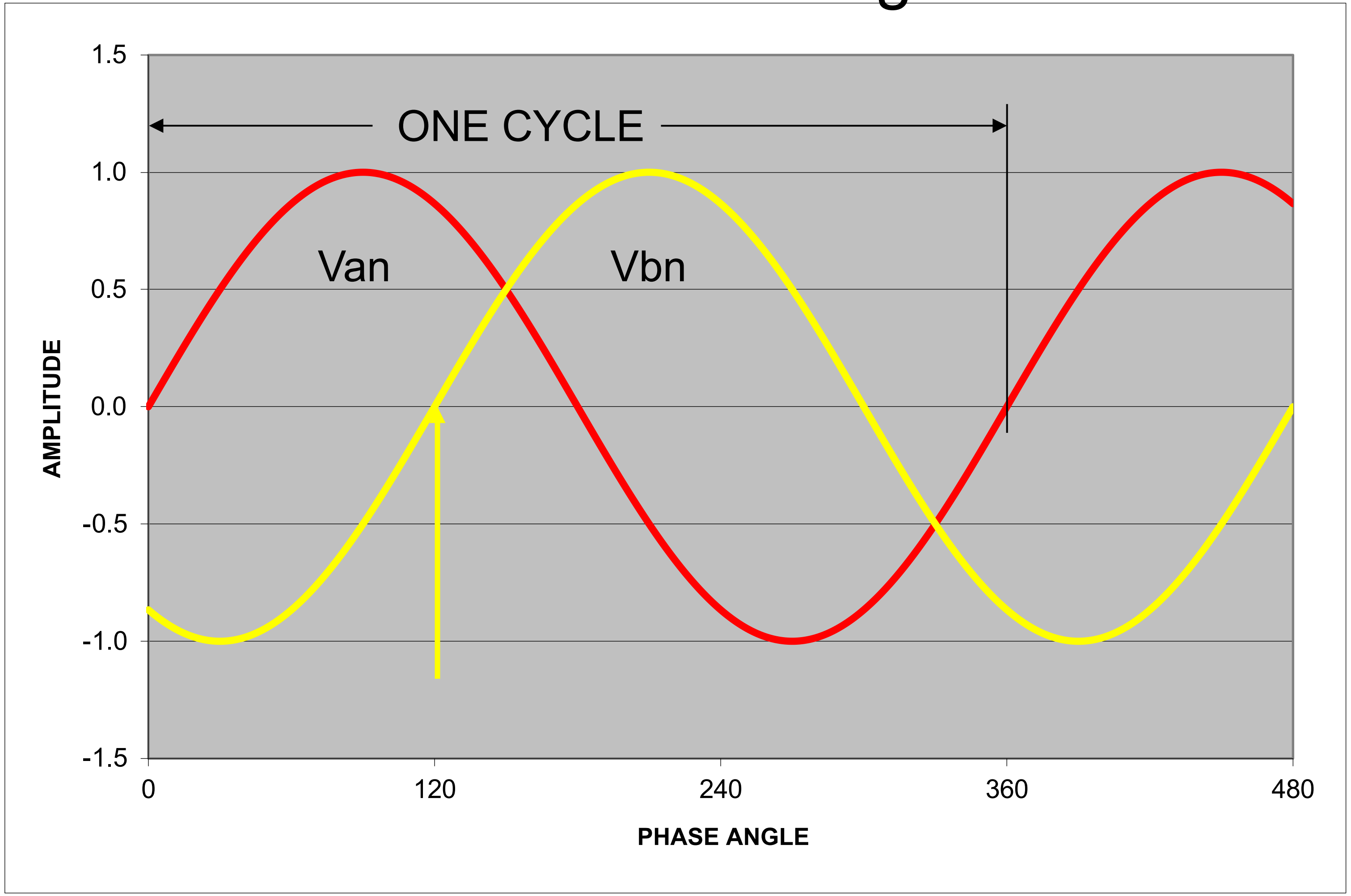
Three Phase Theory

Single Phase - Voltage Plot



Three Phase Theory

Two Phases - Voltage Plot



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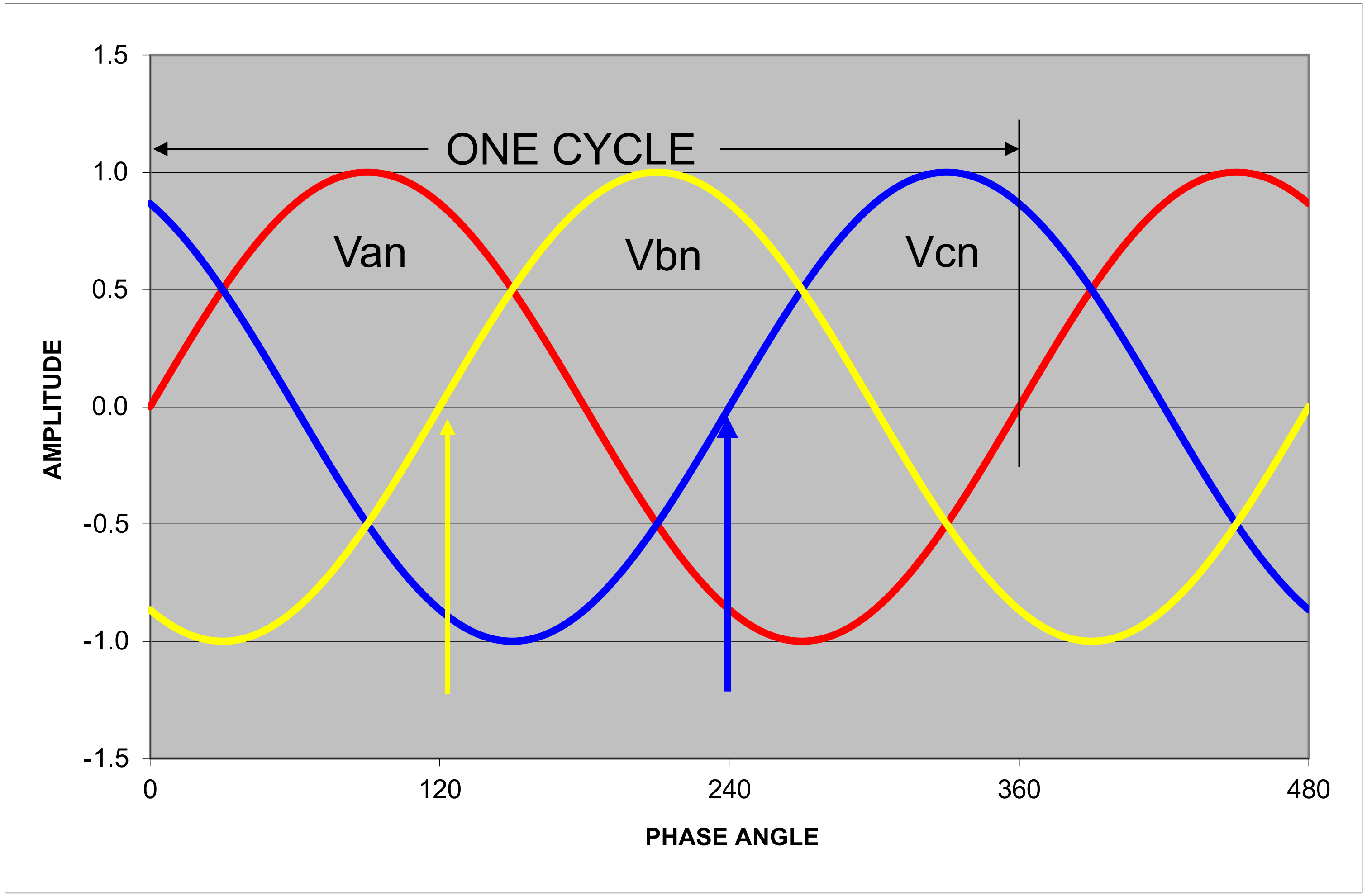
Three Phase - Voltage Plot



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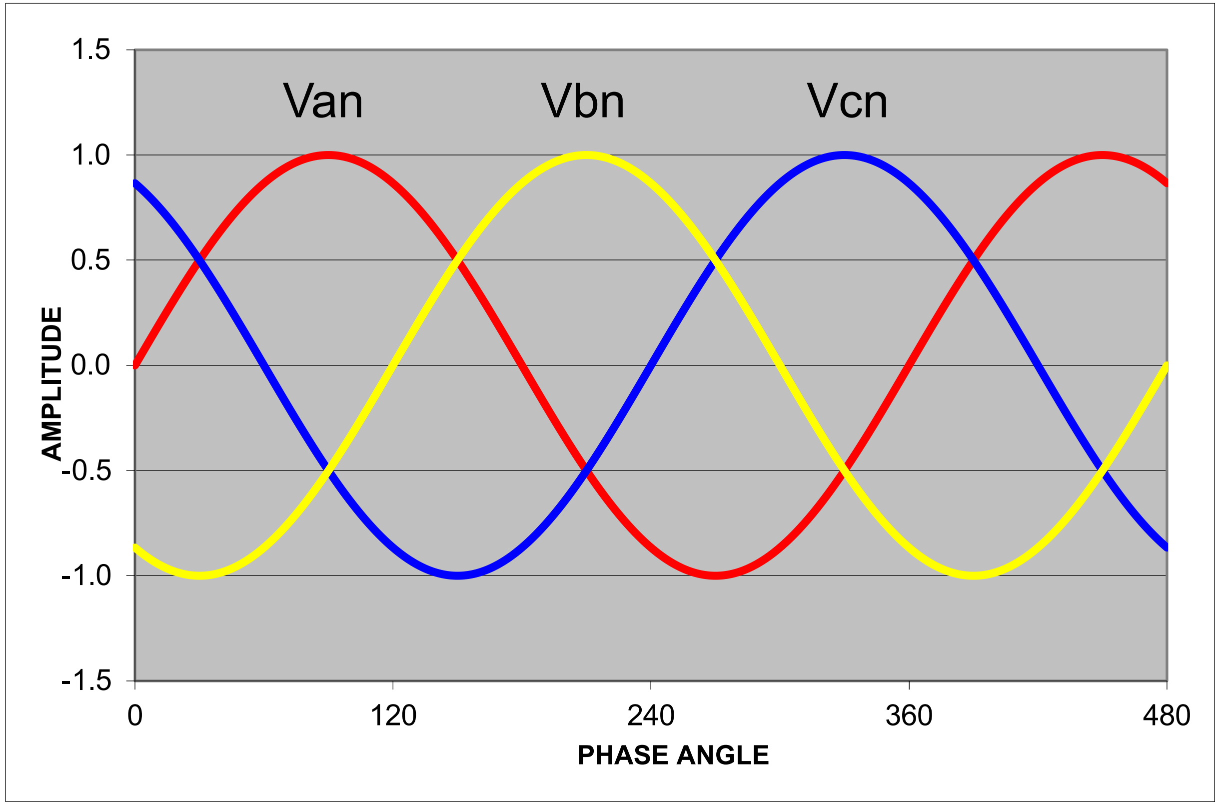
Three Phase Power

At the Generator

Three voltage vectors each separated by 120°.

Peak voltages essentially equal.



Most of what makes three phase systems seem complex is what we do to this simple picture in the delivery system and loads.

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Three Phase Power

Basic Concept – Phase Rotation

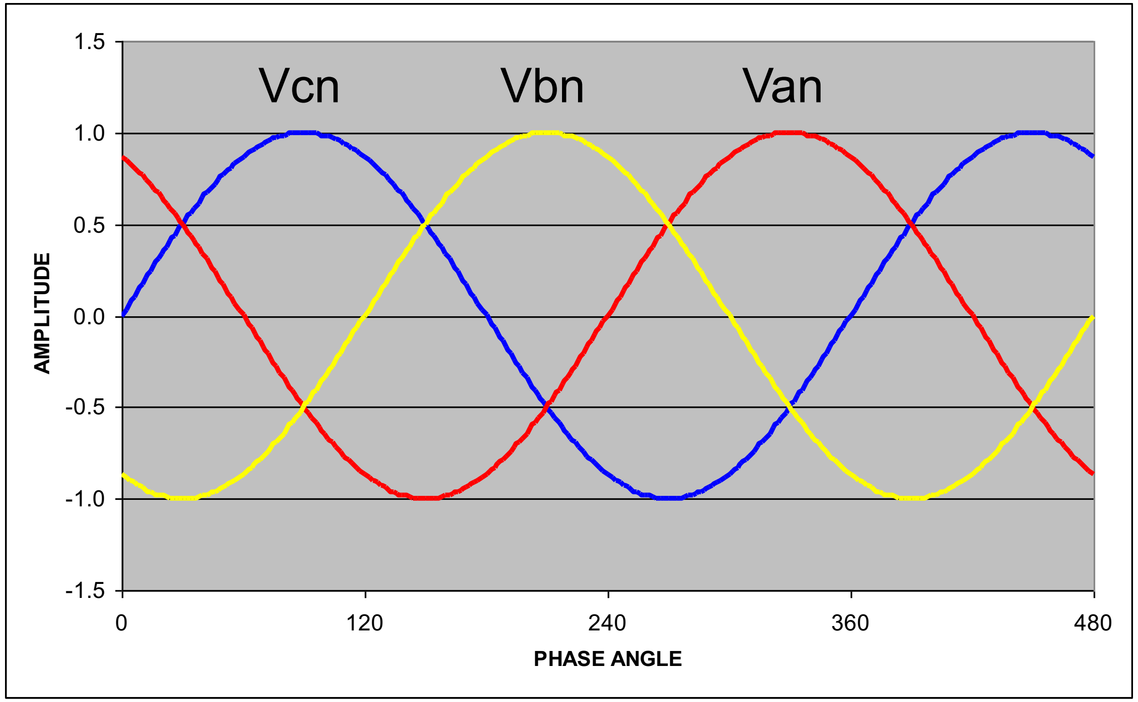
Phase Rotation:

The order in which the phases reach peak voltage.

There are only two possible sequences:

A-B-C (previous slide)

C-B-A (this slide)



Phase rotation is important because the direction of rotation of a three phase motor is determined by the phase order.

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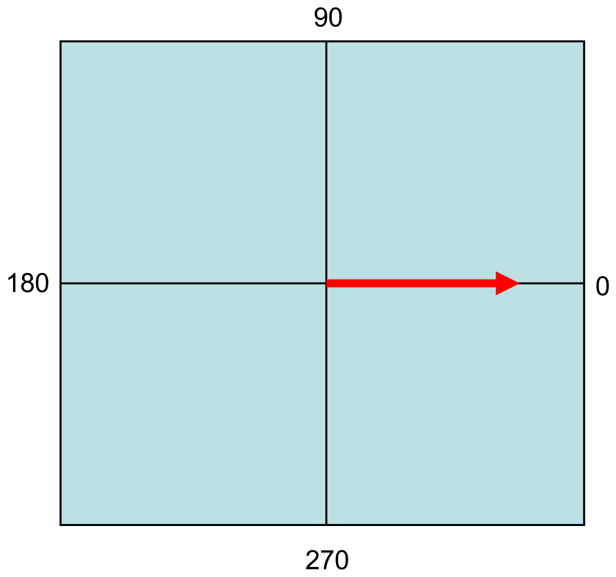
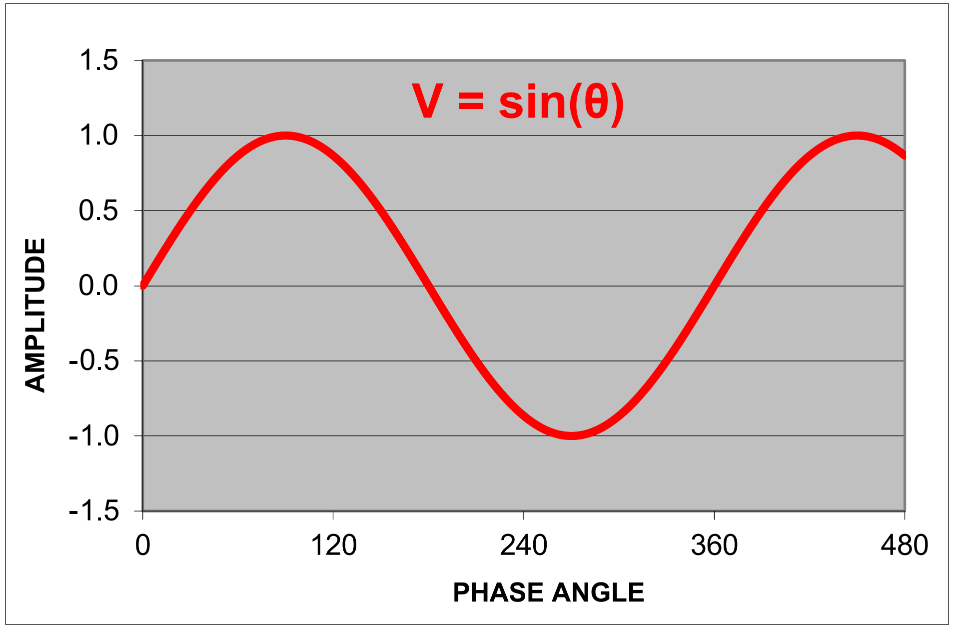
Three Phase Theory

Phasors and Vector Notation

- Phasors are a graphical means of representing the amplitude and phase relationships of voltages and currents.





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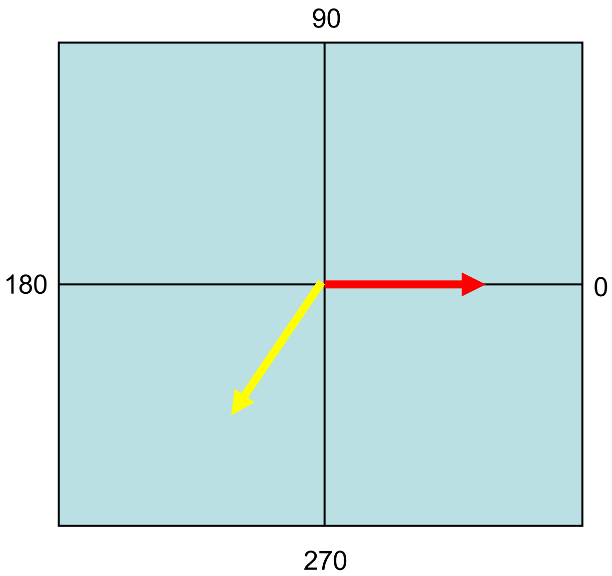
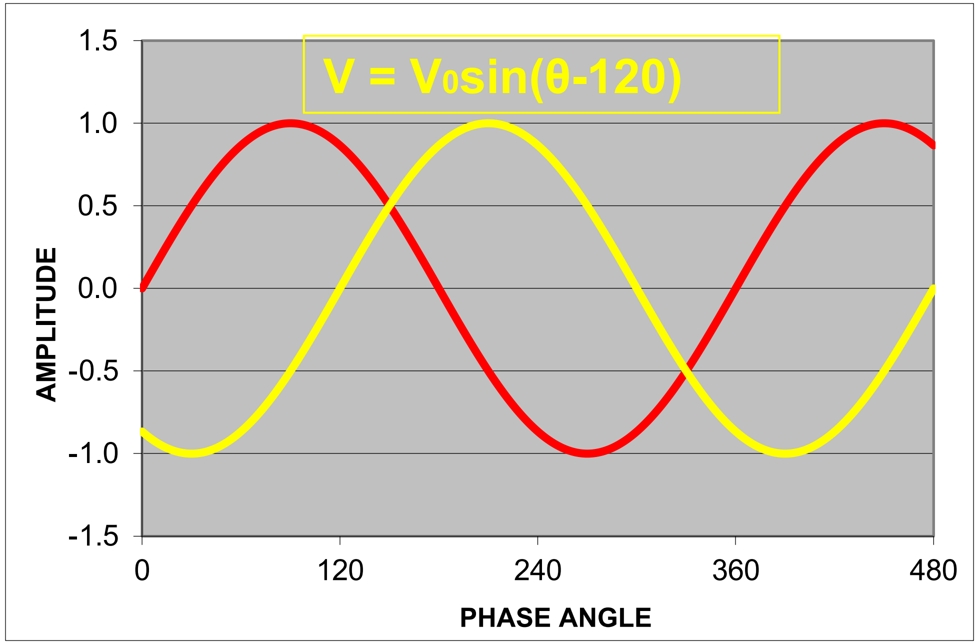
Three Phase Power

Phasors and Vector Notation

- As stated in the Handbook of Electricity Metering, by common consent, counterclockwise phase rotation has been chosen for general use in phasor diagrams.





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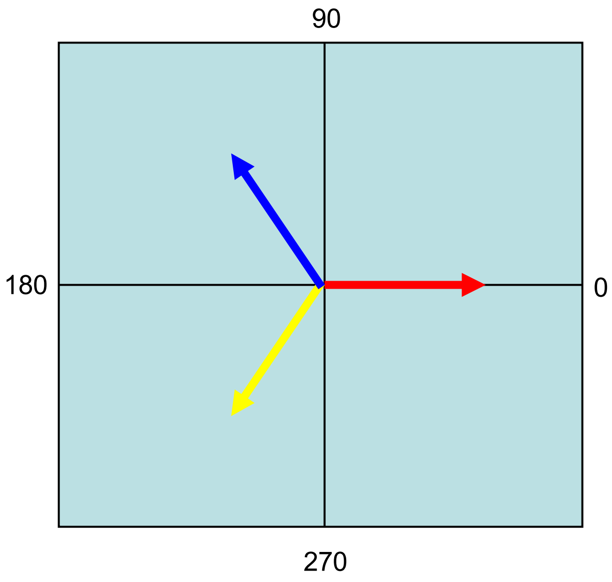
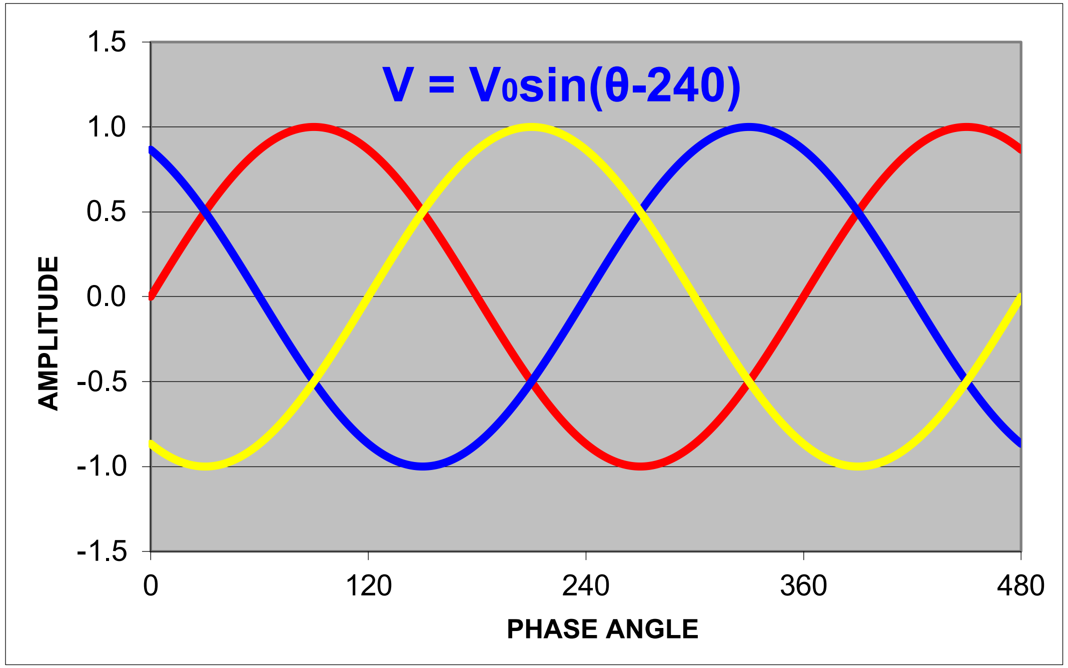
Three Phase Power
Phasors and Vector Notation

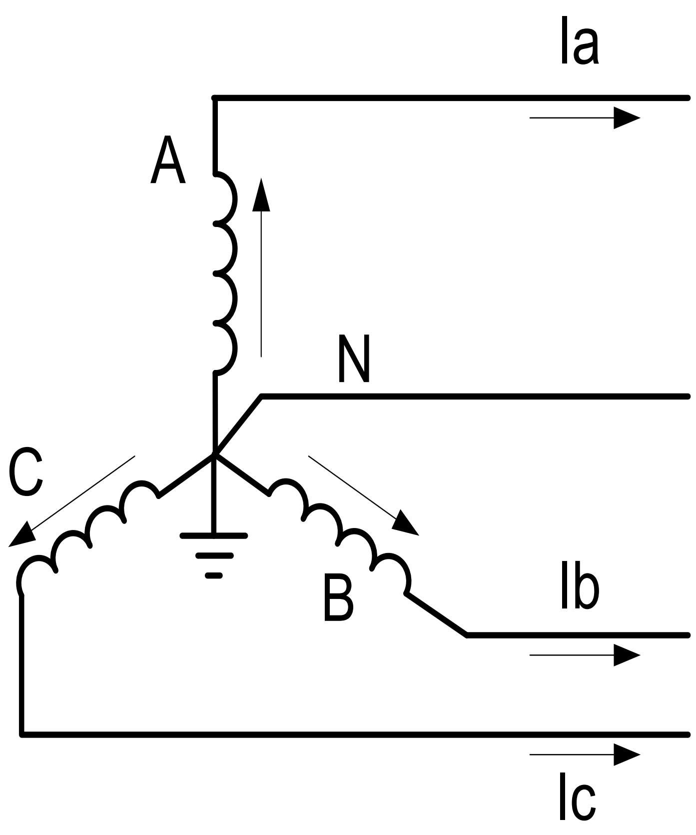
- The phasor diagram for a simple 3-phase system has three voltage phasors equally spaced at 120° intervals.
- Going clockwise the order is A – B – C.





Three Phase Theory
Symbols and Conventions

- Systems formed by interconnecting secondaries of 3 single phase transformers.
- Generally primaries are not show unless details of actual transformer are being discussed.



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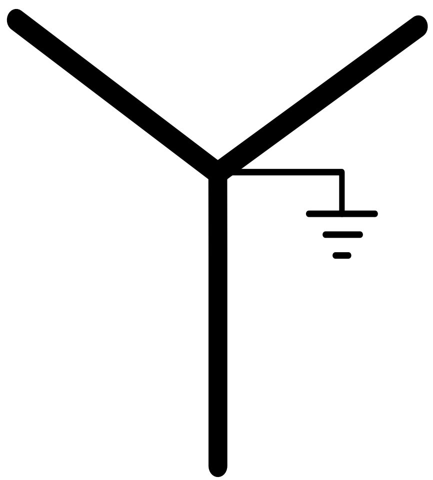
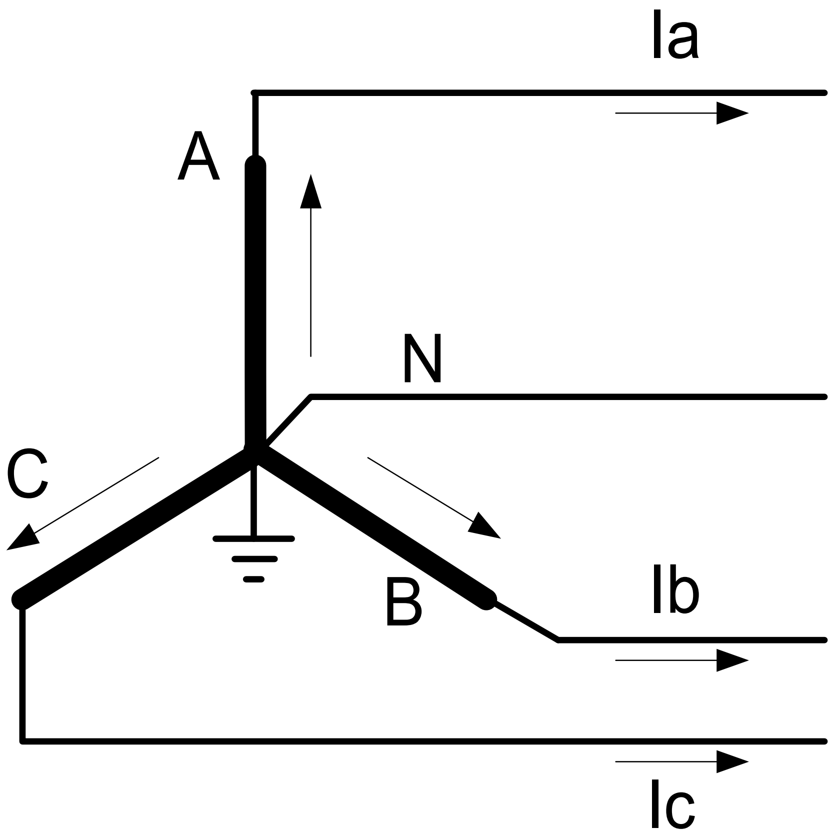
Three Phase Theory

Symbols and Conventions

- Often even the coils are not shown but are replaced by simple line drawings





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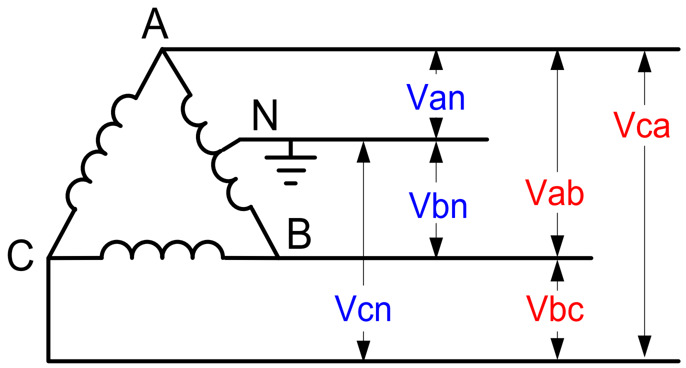
Symbols and Conventions

Labeling

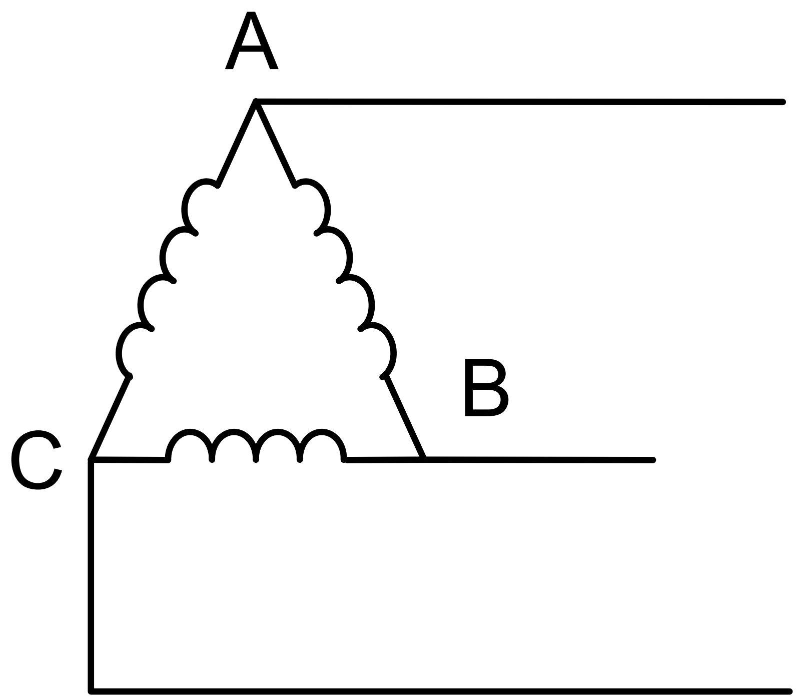
- Voltages are generally labeled Va, Vb, Vc, Vn for the three phases and neutral
- This can be confusing in complex cases.
- The recommended approach is to use two subscripts so the two points between which the voltage is measured are unambiguous.

Vab means voltage at “a” as measured relative to “b”.

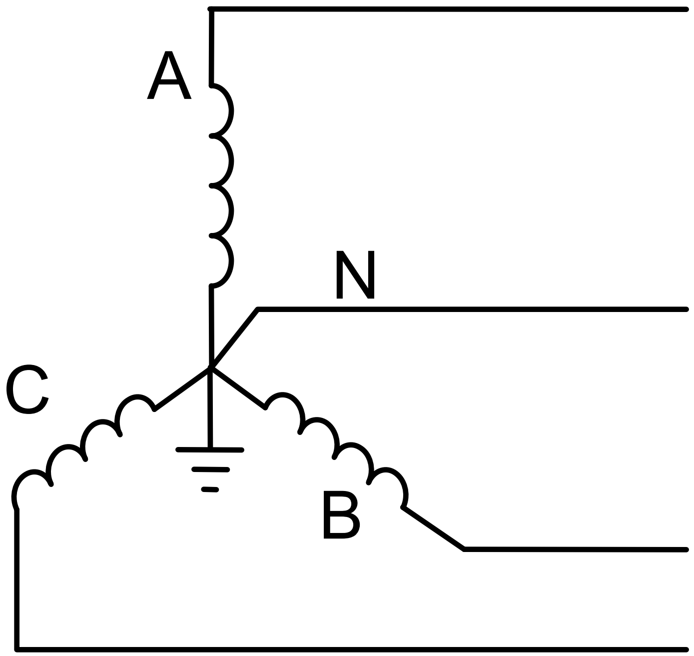


Three Phase Transformers

Delta vs Wye



Delta



Wye

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Three Phase Transformers

Delta vs Wye

- Delta is commonly used for power transmission as it only requires three lines
- Delta is better for a balanced load like a motor and has greater reliability if a winding failure occurs
- Wye offers two voltages – line to neutral and line to line
- Wye is used when a single phase load is required

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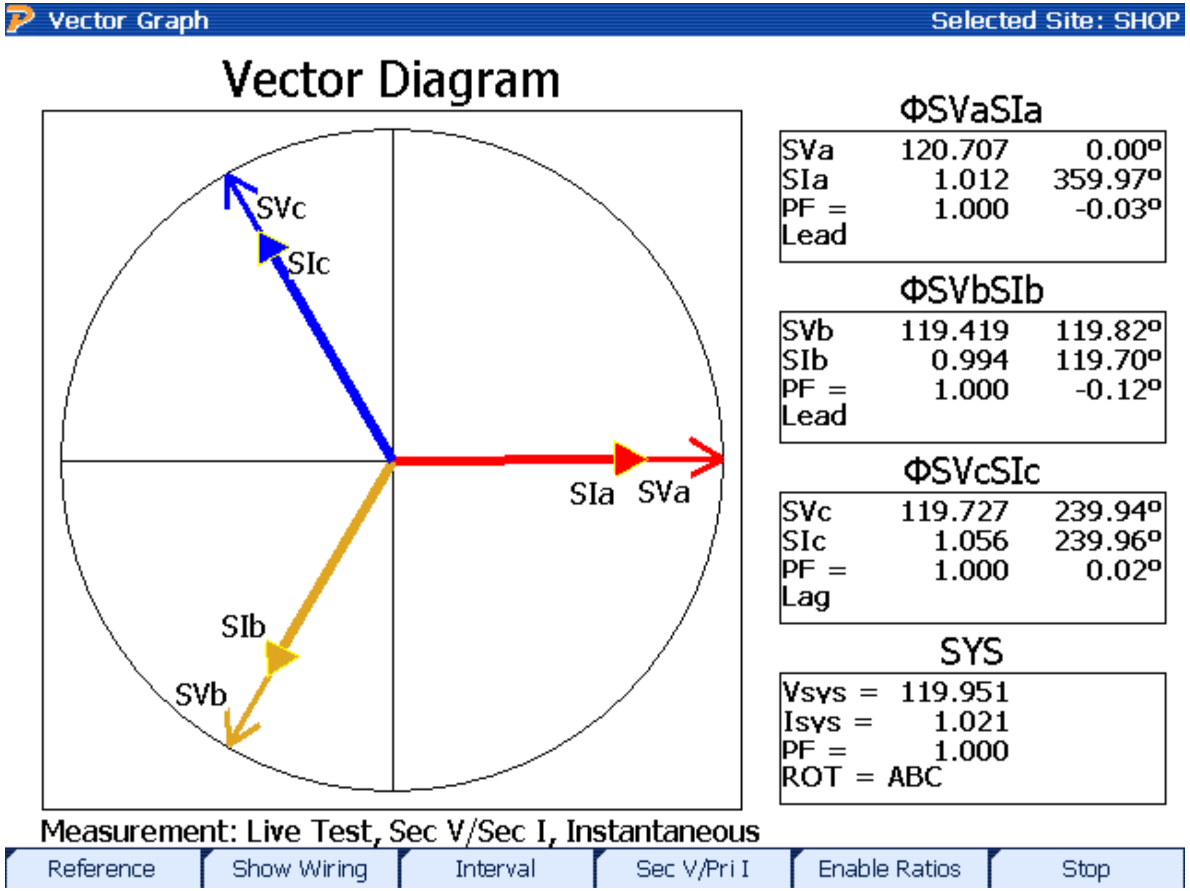
3 Phase, 4-Wire “Y” Service

0° = Unity Power Factor

- Three Voltage Phasors
- 120° Apart
- Three Current Phasors
- Aligned with Voltage at PF=1



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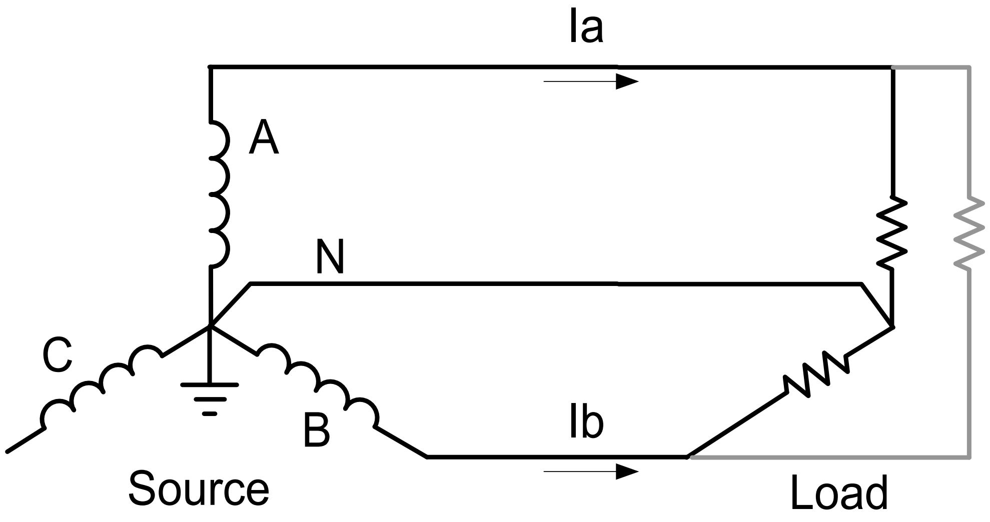
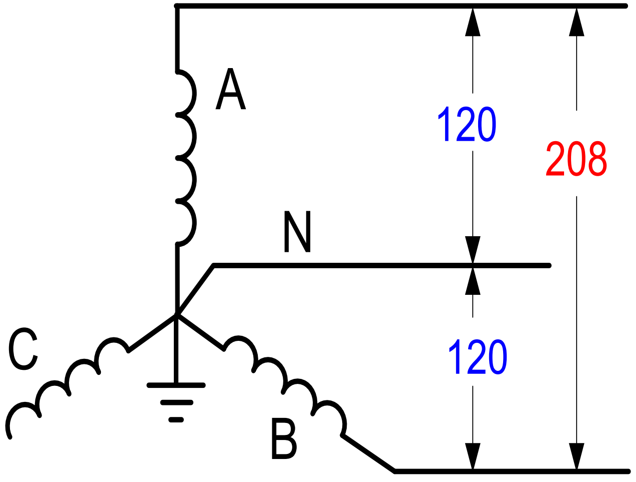
2 Phase, 3-Wire "Y" Service

"Network Connection"

Single phase variant of the service.





Two voltage sources with their returns connected to a common point.

Provides 208 rather than 240 volts across "high side" wires.

$$V_{AB} = \sqrt{3} V_{AN}$$

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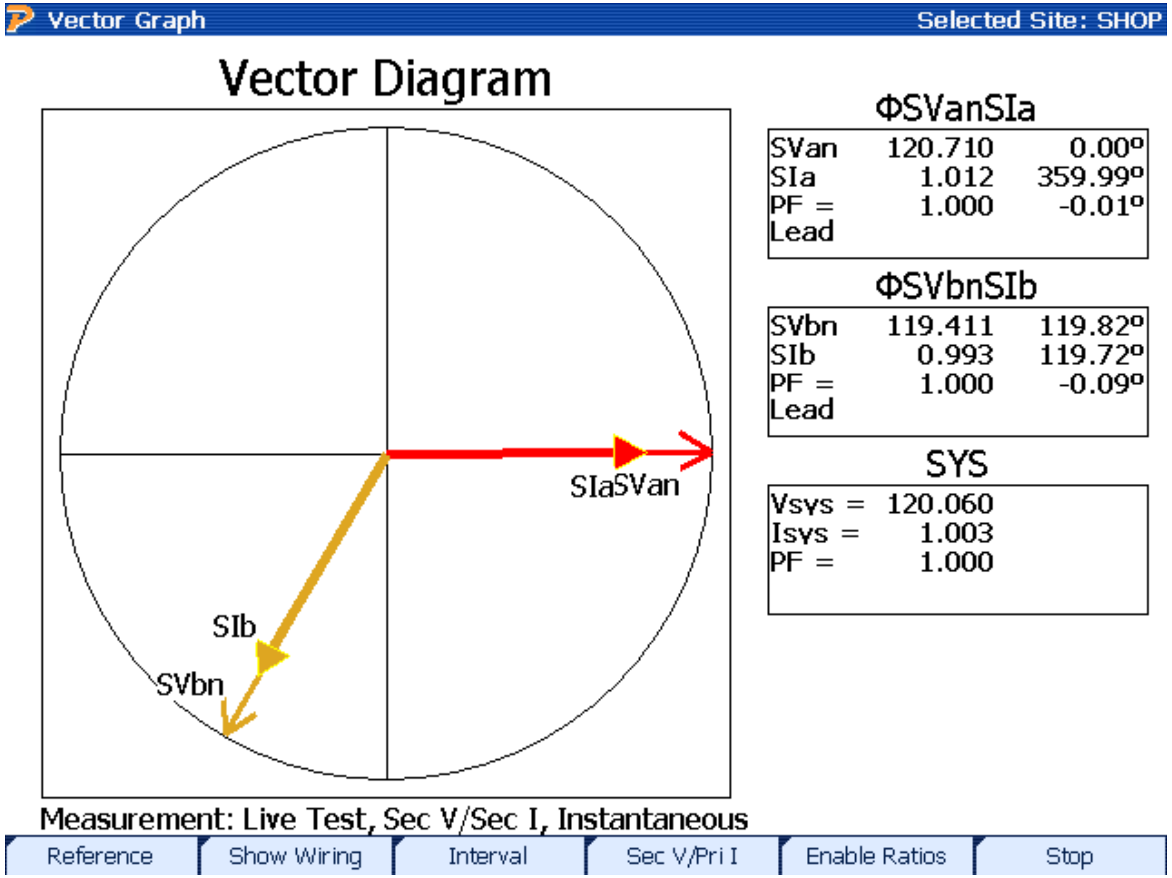
2 Phase, 3-Wire "Network" Service

- Two Voltage Phasors
- 120° Apart
- Two Current Phasors
- Aligned with Voltage at PF=1



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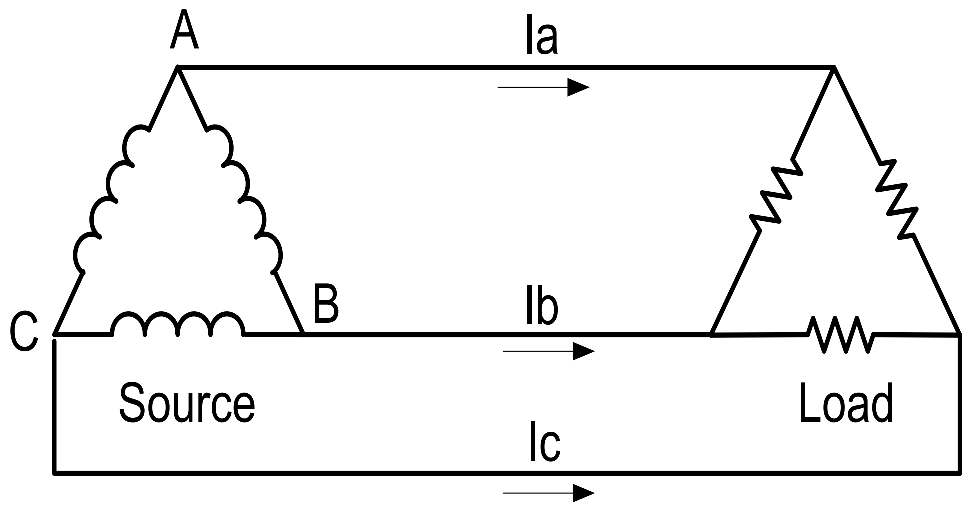
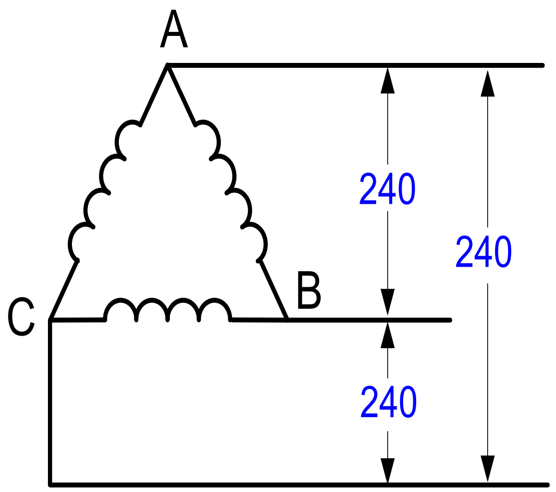
3 Phase, 3-Wire Delta Service

Common service type for industrial customers. This service has NO neutral.



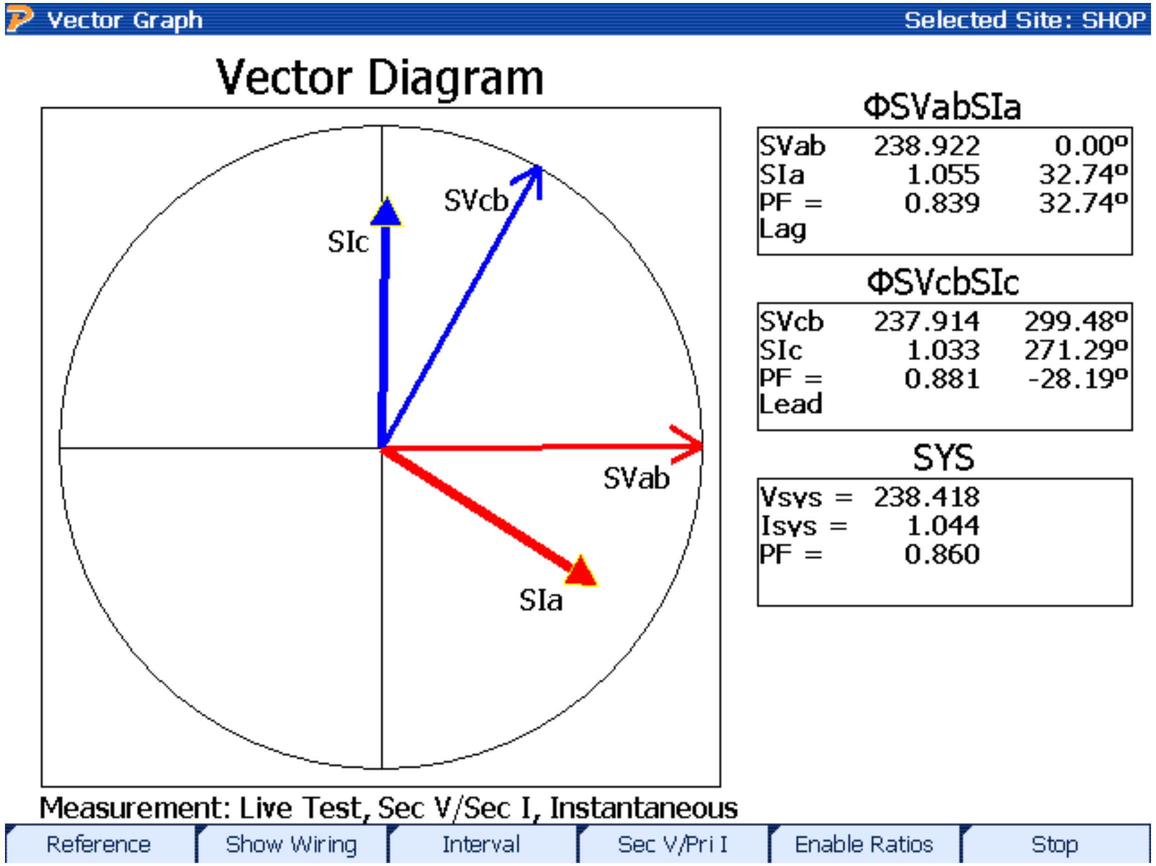


- Voltages normally measured relative to phase B.
- Voltage and current vectors do not align.
- Service is provided even when a phase is grounded.

3 Phase, 3-Wire Delta Service

Resistive Loads

- Two Voltage Phasors
- 60° Apart
- Two Current Phasors
- For a resistive load one current leads by 30° while the other lags by 30°



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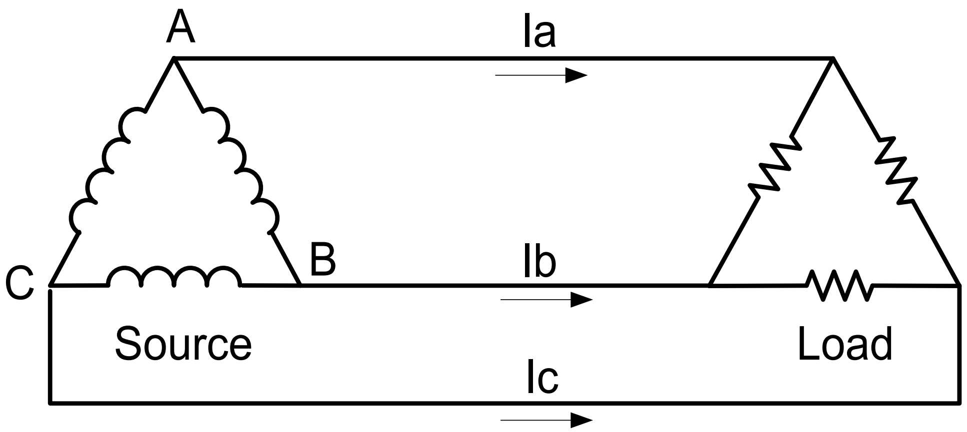
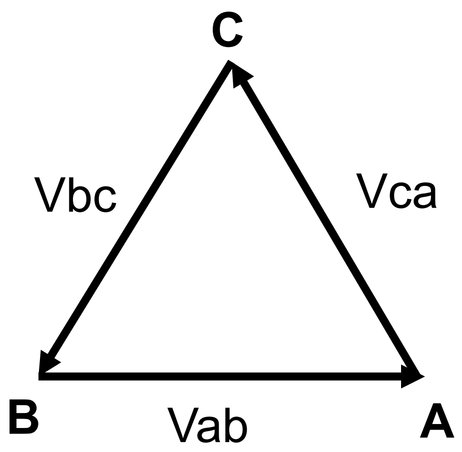
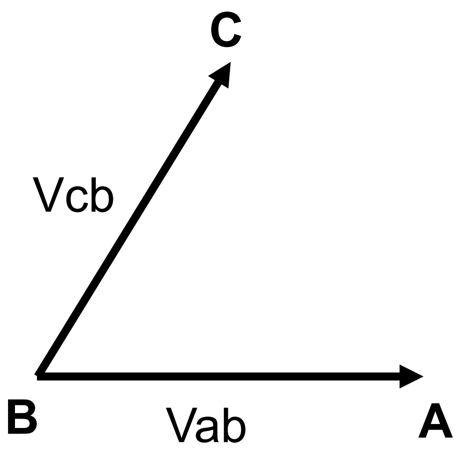
3 Phase, 3-Wire Delta Service

Understanding the Diagram







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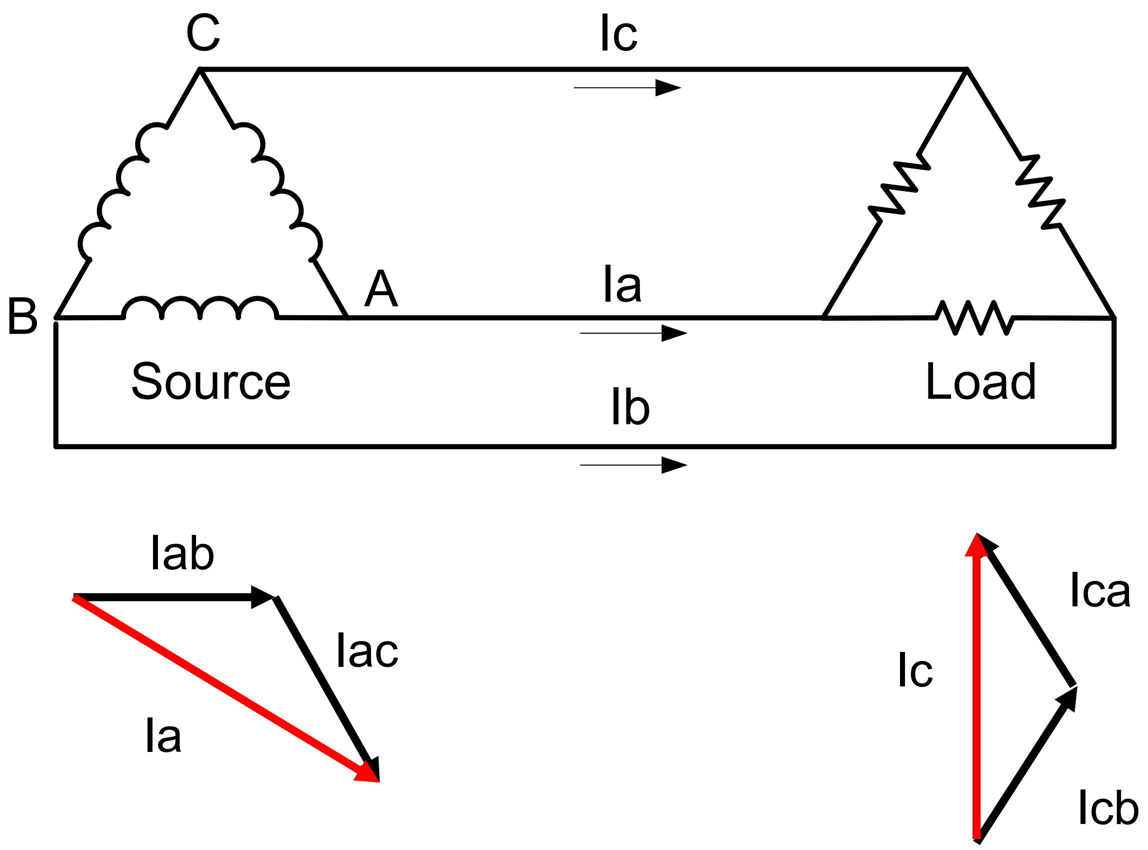
3 Phase, 3-Wire Delta Service

Understanding the Diagram



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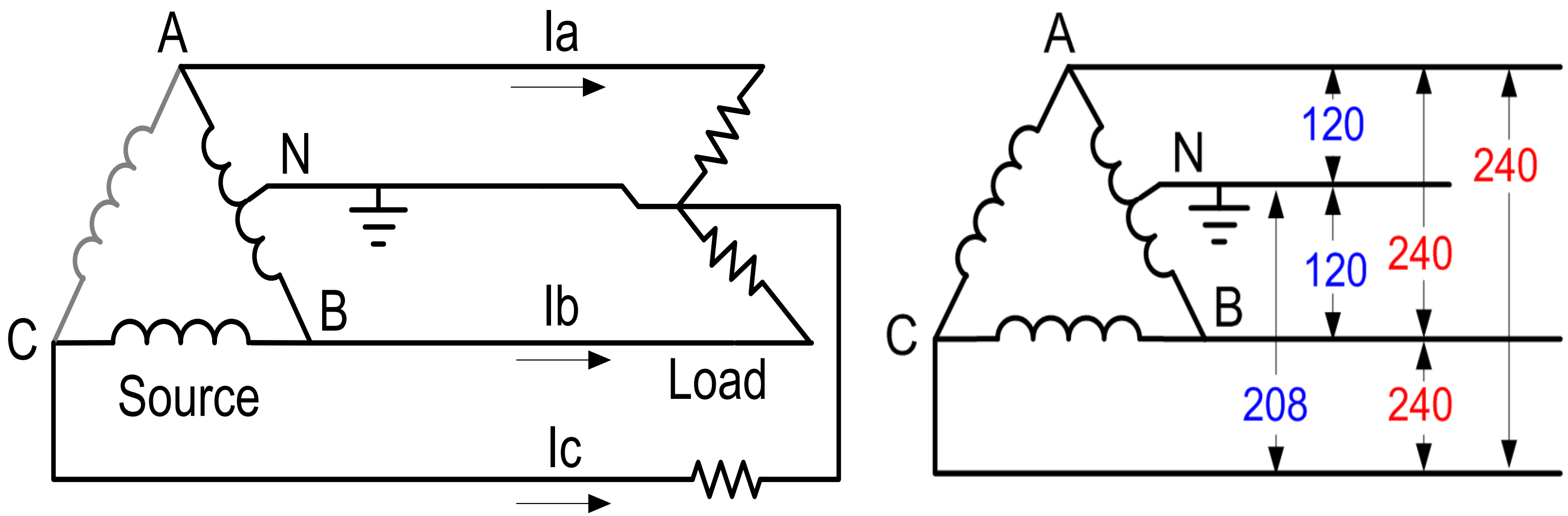
3 Phase, 4-Wire Delta Service

Common service type for industrial customers. Provides a residential like 120/240 service (lighting service) single phase 208 (high side) and even 3 phase 240 V.



•Voltage phasors form a “T” 90° apart

•Currents are at 120° spacing

•In 120/120/208 form only the “hot” (208) leg has its voltage and current vectors aligned.

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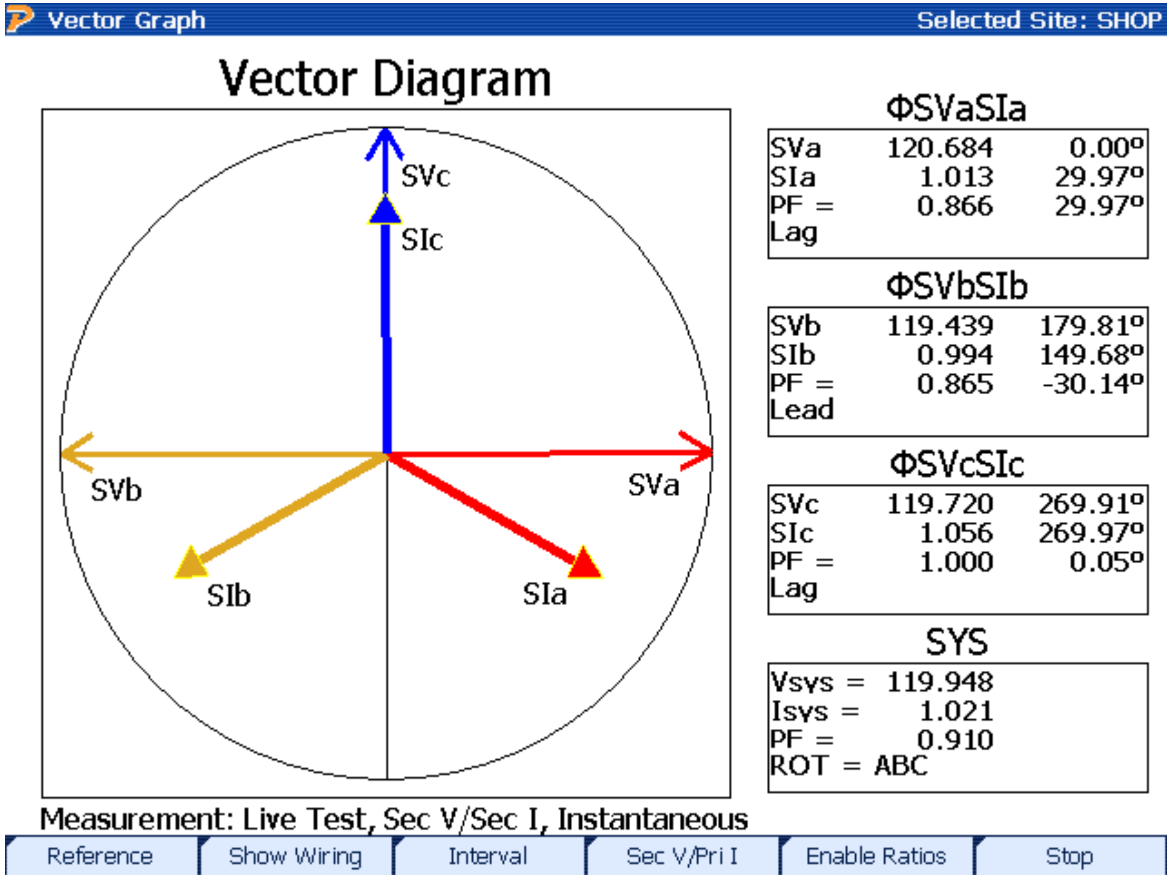
3 Phase, 4-Wire Delta Service

Resistive Load

- Three Voltage Phasors
- 90° Apart
- Three Current Phasors
- 120° apart



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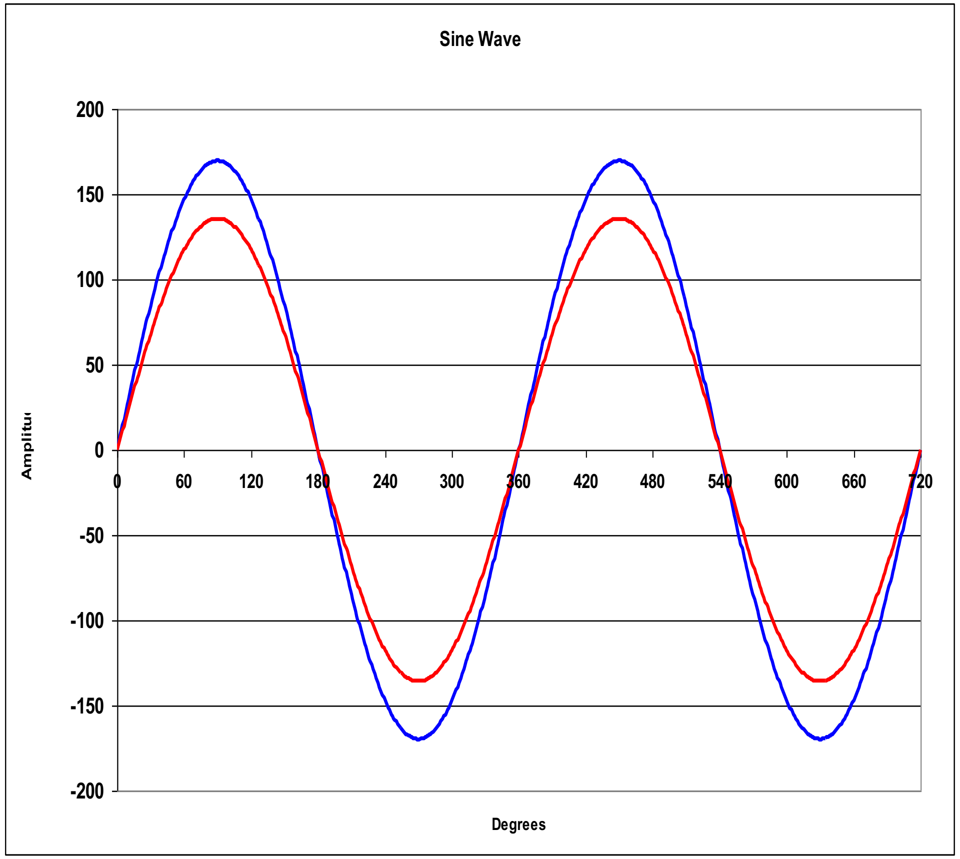
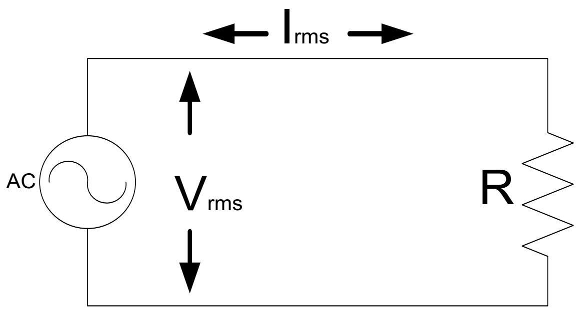
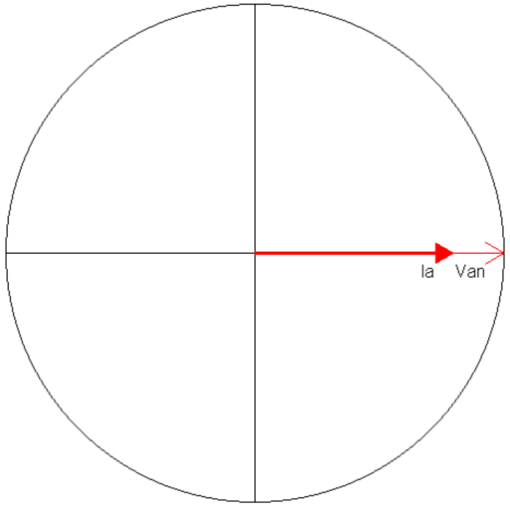
AC Theory – Resistive Load







Resistors are measured in Ohms. When an AC voltage is applied to a resistor, the current is in phase. A resistive load is considered a “linear” load because when the voltage is sinusoidal the current is also sinusoidal.

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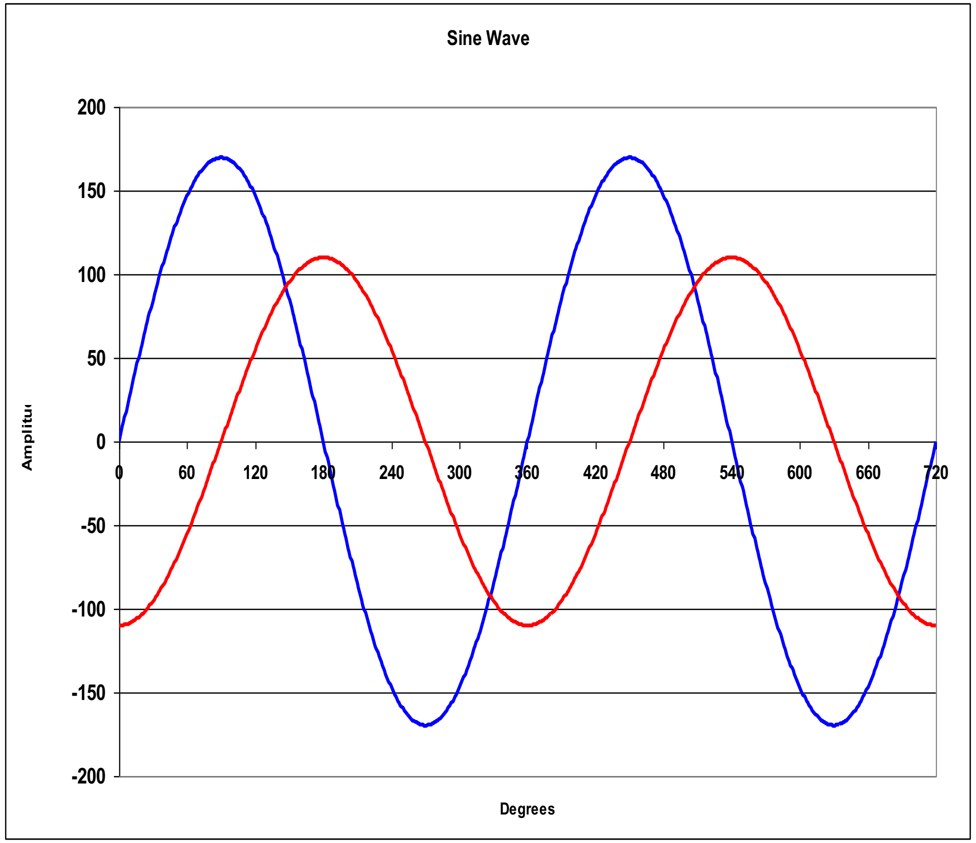
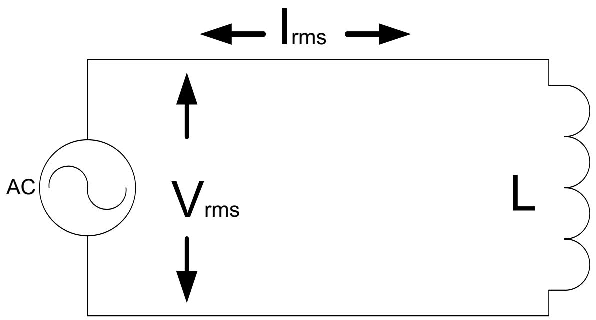
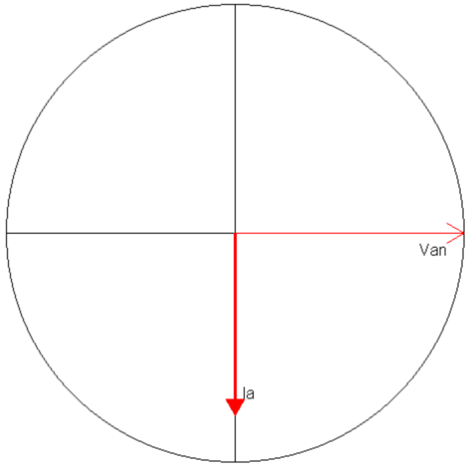
AC Theory – Inductive Load







Inductors are measured in Henries. When an AC voltage is applied to an inductor, the current is 90 degrees out of phase. We say the current "lags" the voltage. A inductive load is considered a "linear" load because when the voltage is sinusoidal the current is also sinusoidal.

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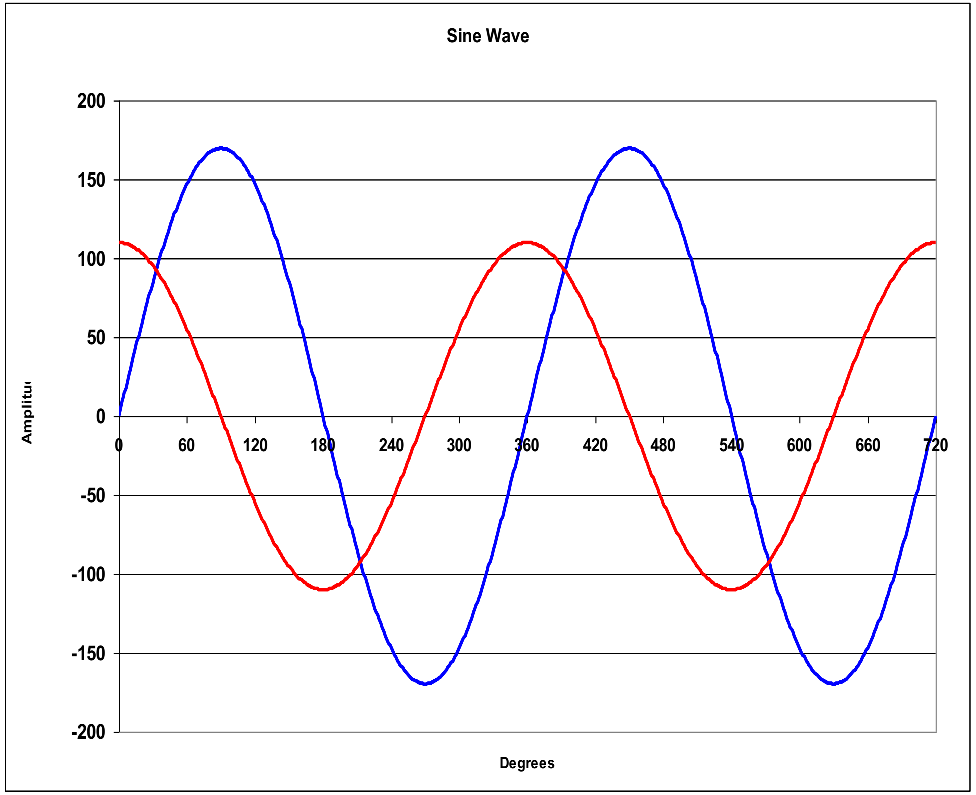
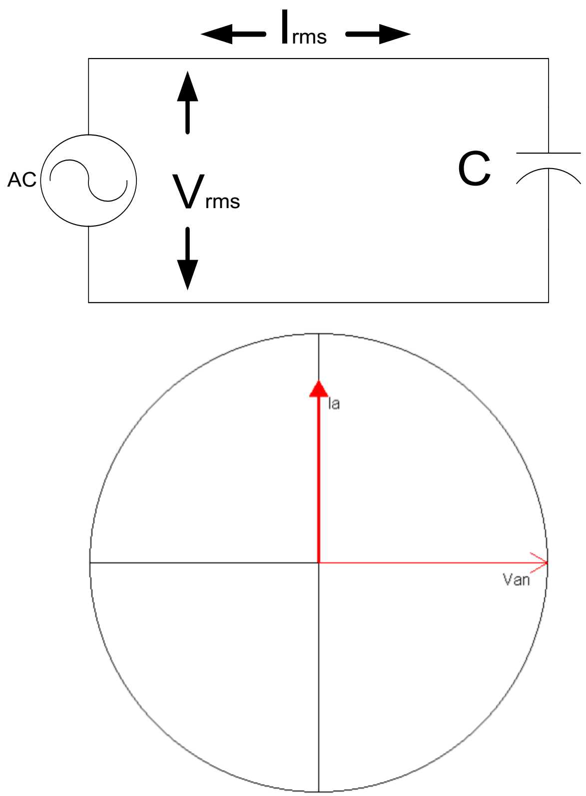
AC Theory – Capacitive Load





Capacitors are measured in Farads. When an AC voltage is applied to a capacitor, the current is 90 degrees out of phase. We say the current "leads" the voltage. A capacitive load is considered a "linear" load because when the voltage is sinusoidal the current is sinusoidal.

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AC Theory – Power

- Power is defined as P = VI
- Since the voltage and current at every point in time for an AC signal is different, we have to distinguish between instantaneous power and average power. Generally when we say “power” we mean average power.
- Average power is only defined over an integral number of cycles.

Time Out for Trig

(Right Triangles)

The Right Triangle:

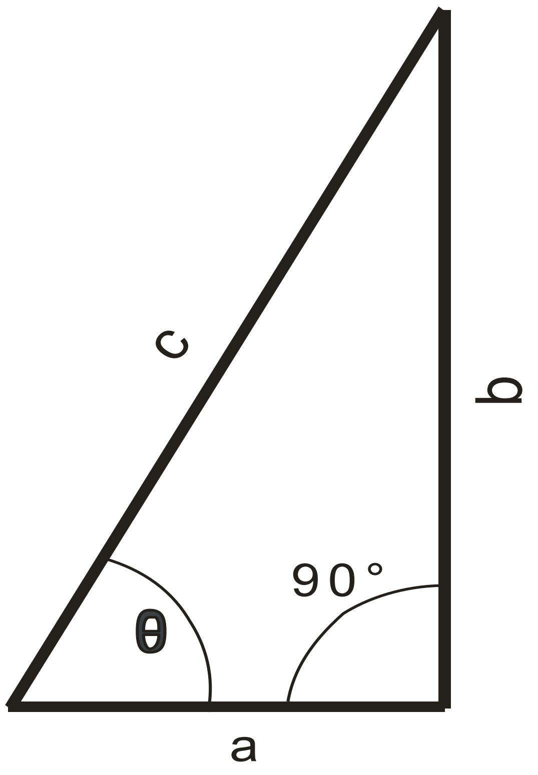
The Pythagorean theory

$c^2 = a^2 + b^2$

$$Sin(\theta) = \frac{b}{c}$$

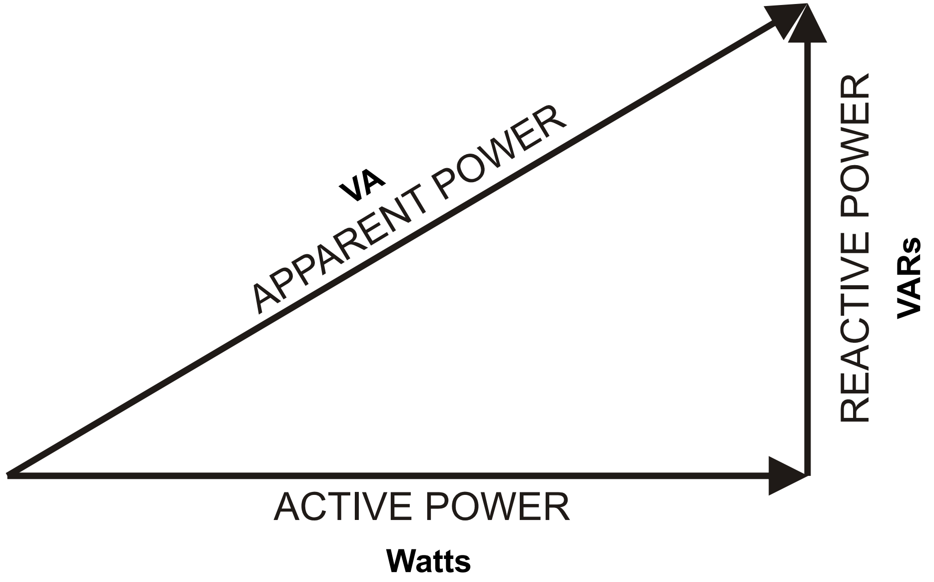
$$Cos(\theta) = \frac{a}{c}$$

$$Tan(\theta) = \frac{b}{a}$$



AC Theory – Power Triangle

(Sinusoidal Waveforms)



If $V = Sin(\omega t)$ and $I = Sin(\omega t - \theta)$ (the load is linear)
then

Active Power =	$VICos(\theta)$	Watts
Reactive Power =	$VISin(\theta)$	VARs
Apparent Power =	VI	VA
Power Factor =	Active/Apparent = $Cos(\theta)$	

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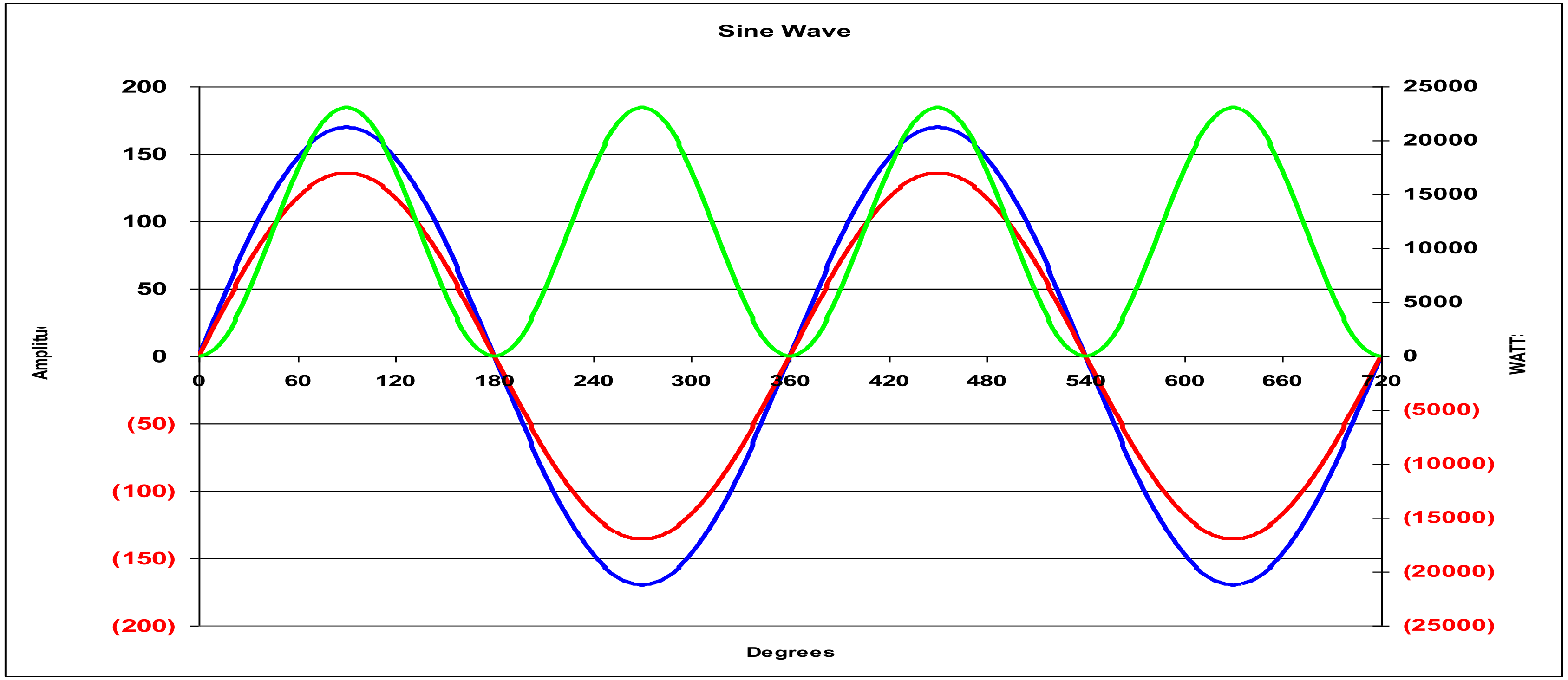
AC Theory

Instantaneous Power

For a resistive load: $p = vi = 2VISin^2(\omega t) = VI(1 - Cos(2\omega t))$



$V = 120\sqrt{2}Sin(2\pi ft)$

$I = 96\sqrt{2}Sin(2\pi ft)$

$P = 23040Sin^2(2\pi ft)$

P = 11520 Watts

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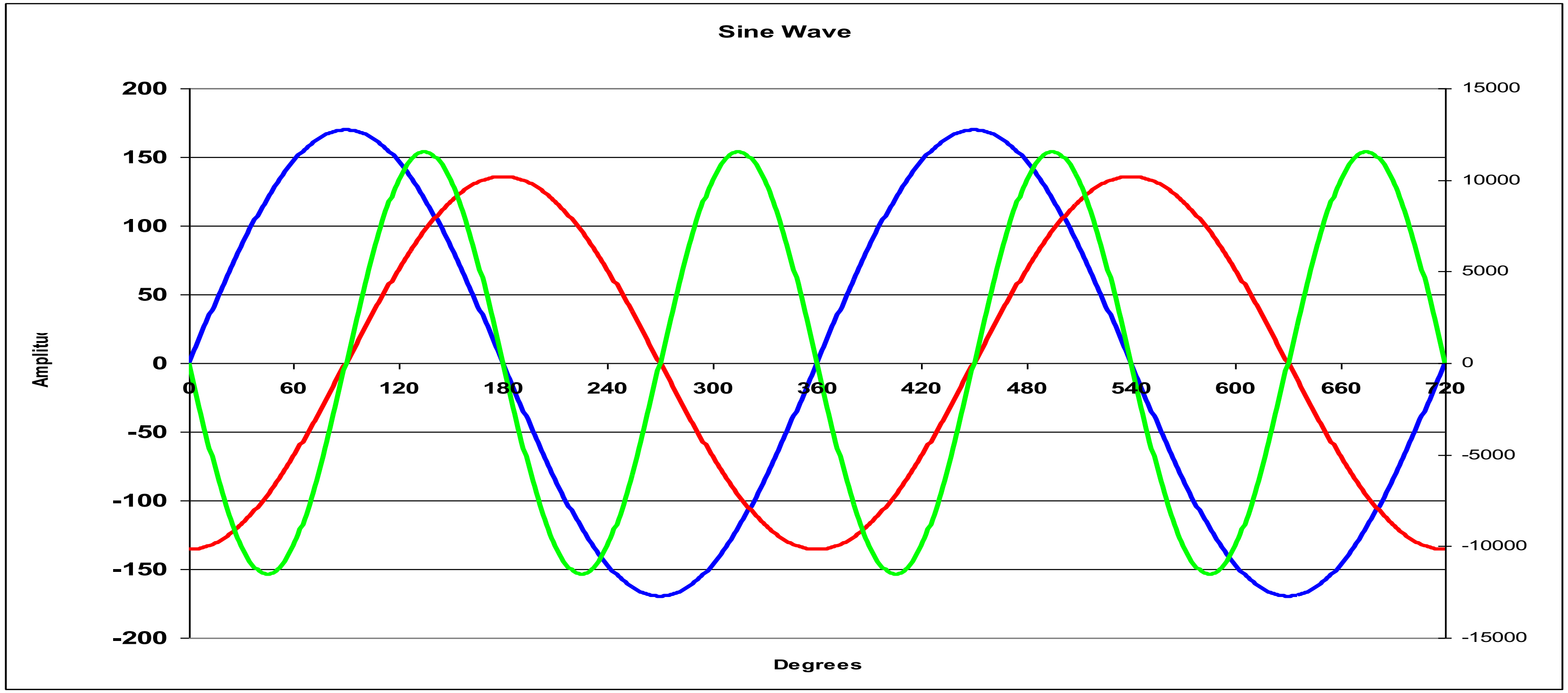
AC Theory

Instantaneous Power

For an inductive load: $p = vi = 2VISin(\omega t)Sin(\omega t - 90) = -VISin(2\omega t)$



$V = 120\sqrt{2}Sin(2\pi ft)$

$I = 96\sqrt{2}Sin(2\pi ft - 90)$

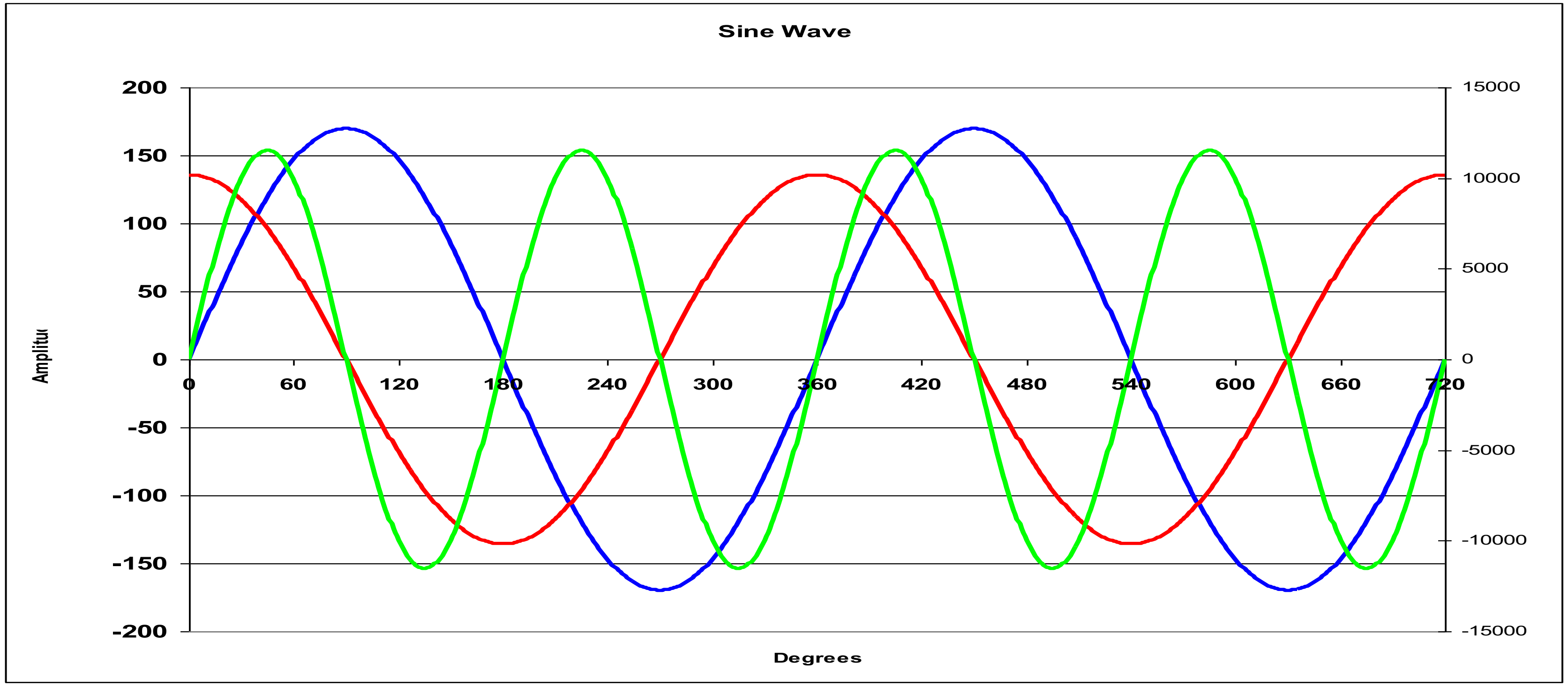
$P = -11520Sin(2\pi ft)$

P = 0 Watts

AC Theory

Instantaneous Power

For a capacitive load: $p = vi = 2VISin(\omega t)Sin(\omega t + 90) = VISin(2\omega t)$



$V = 120\sqrt{2}Sin(2\pi ft)$

$I = 96\sqrt{2}Sin(2\pi ft + 90)$

$P = 11520Sin(2\pi ft)$

P = 0 Watts

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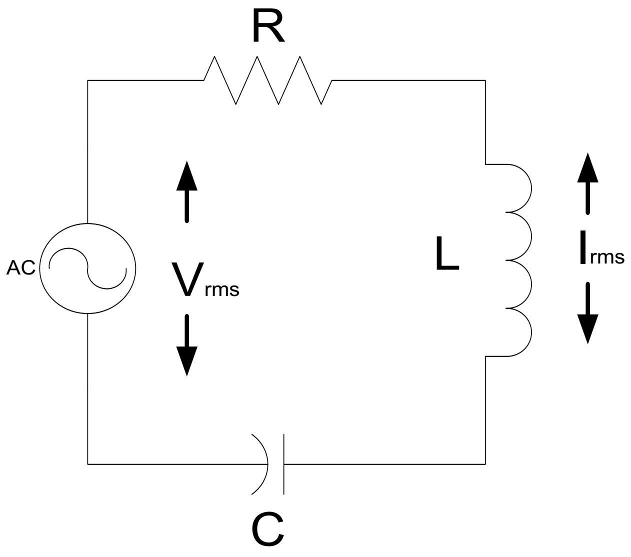
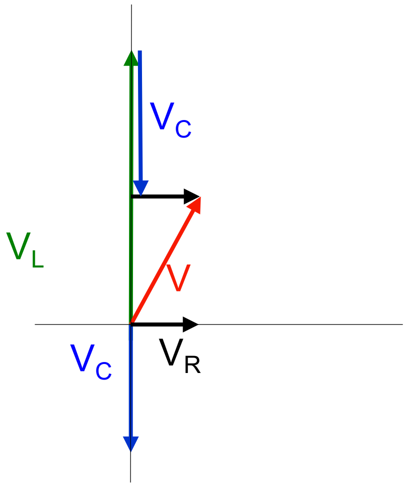
AC Theory – Complex Circuits





Amplitude (Current)

$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Phase (Current)

$$\varphi = ArcTan\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]$$

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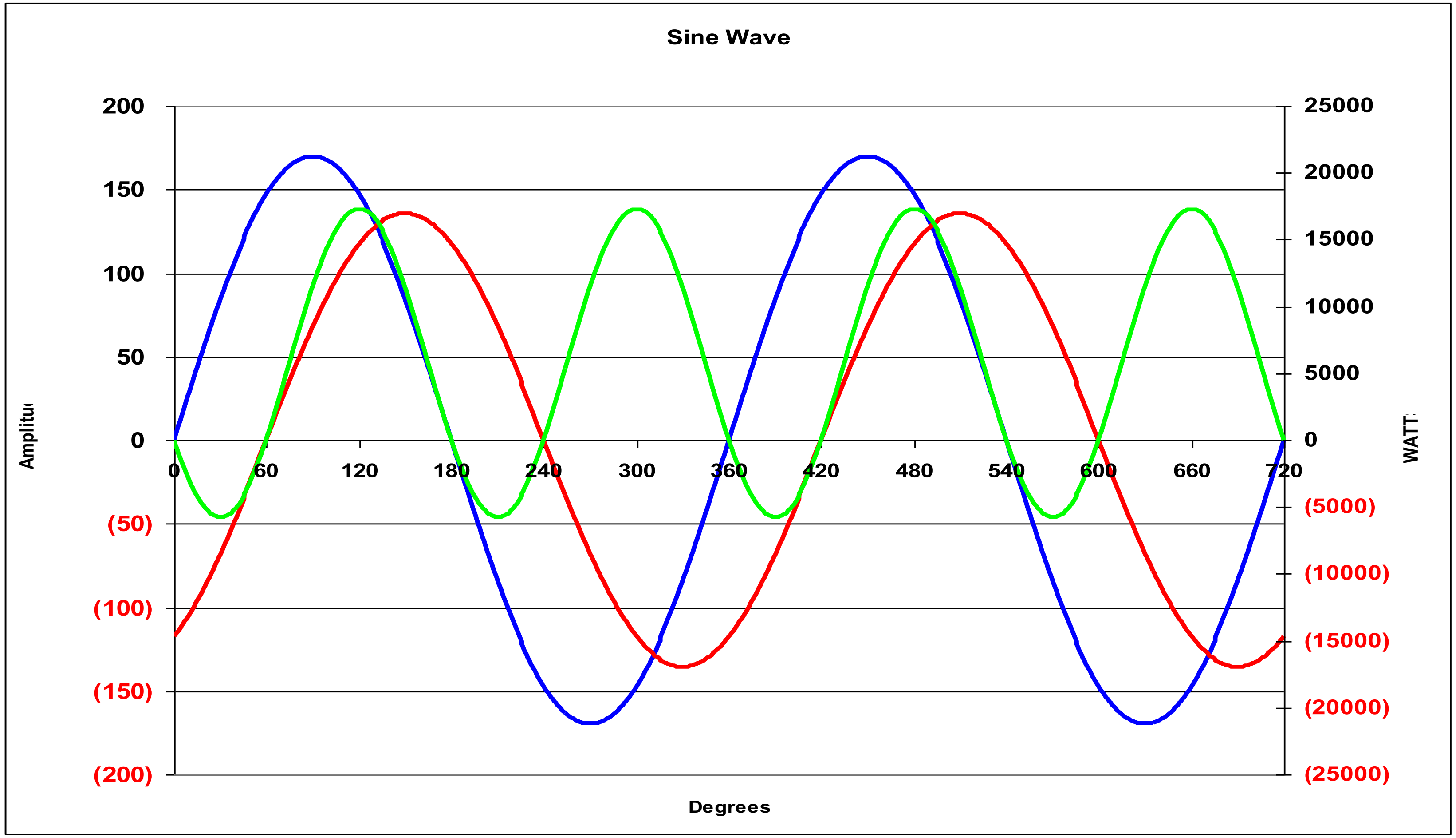
AC Theory – Instantaneous Power



$$V = 120\sqrt{2}Sin(2\pi ft)$$

$$I = 96\sqrt{2}Sin(2\pi ft - 60)$$

$$P = VI = 23040(Cos(60°) + Cos(4\pi ft - 60°)) = 19953 - 23040Cos(4\pi ft - 60°)$$

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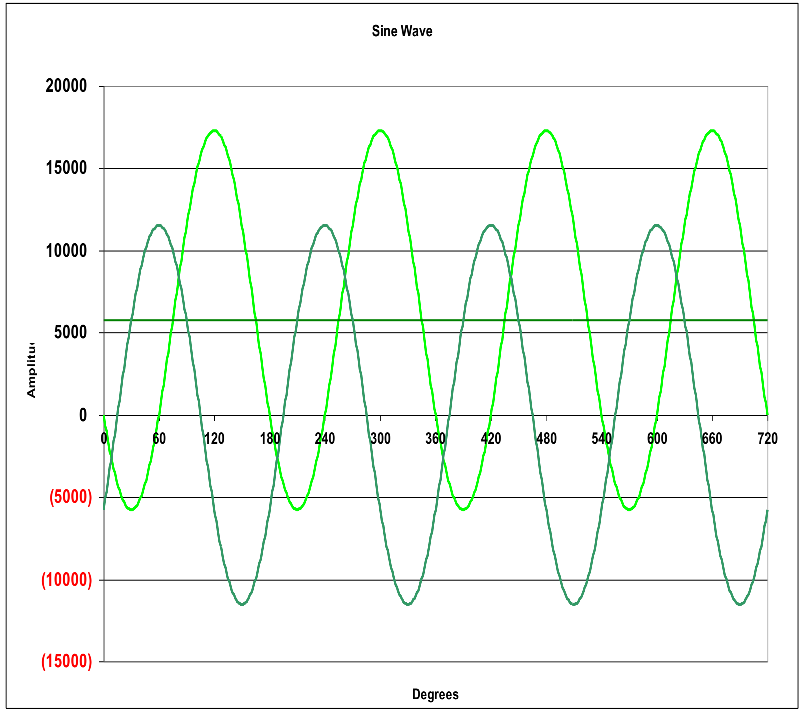
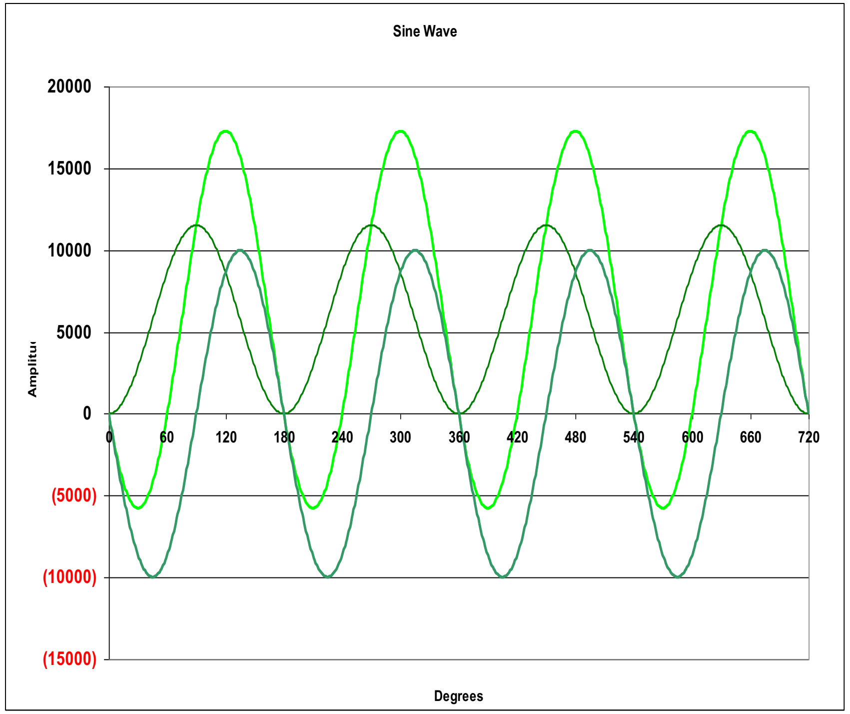
AC Theory – Instantaneous Power

- From IEEE1459 instantaneous power can be written in several forms:





Active Power

Reactive Power

$$p = VI\cos\theta - VI\cos(2\omega t - \theta)$$

$$p = VI\cos\theta[1 - \cos(2\omega t)] - VI\sin\theta\sin(2\omega t)$$

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Three Phase Power

Blondel's Theorem

If energy be supplied to any system of conductors through N wires, the total power in the system is given by the algebraic sum of the readings of N wattmeters, so arranged that each of the N wires contains one current coil, the corresponding voltage coil being connected between that wire and some common point. If this common point is on one of the N wires, the measurement may be made by the use of N-1 wattmeters.

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Three Phase Power

Blondel's Theorem

- Simply – We can measure the power in a N wire system by measuring the power in N-1 conductors.
- For example, in a 4-wire, 3-phase system we need to measure the power in 3 circuits.

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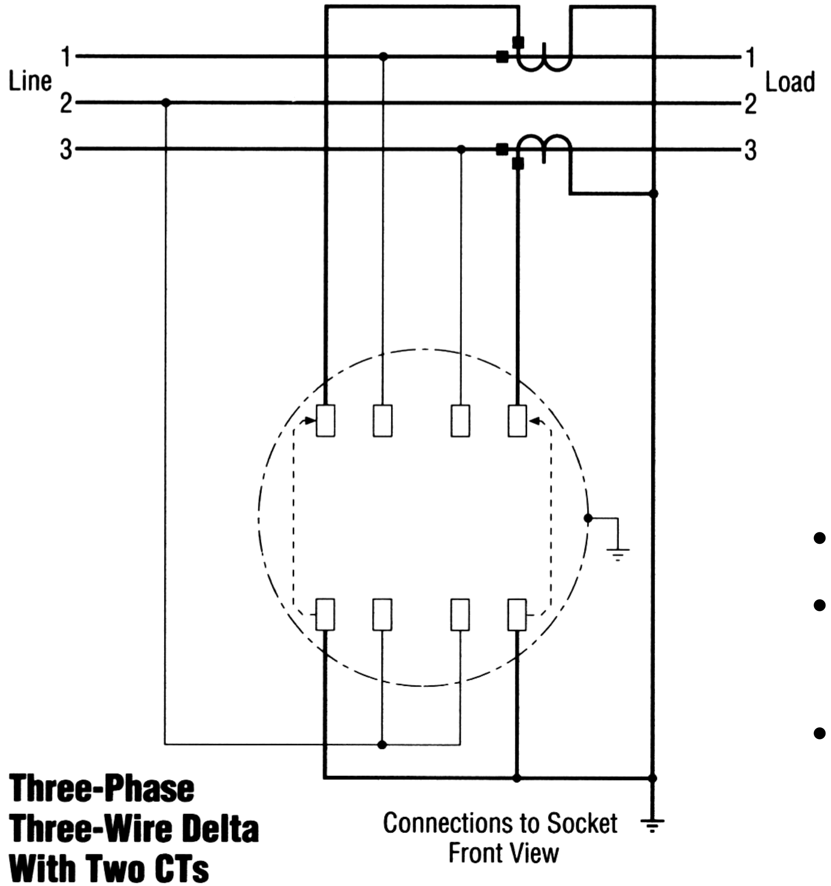
Three Phase Power

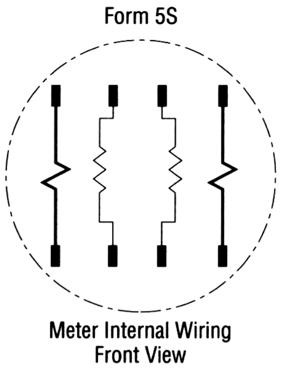
Blondel's Theorem

- If a meter installation meets Blondel's Theorem then we will get accurate power measurements under all circumstances.
- If a metering system does not meet Blondel's Theorem then we will only get accurate measurements if certain assumptions are met.

Blondel's Theorem



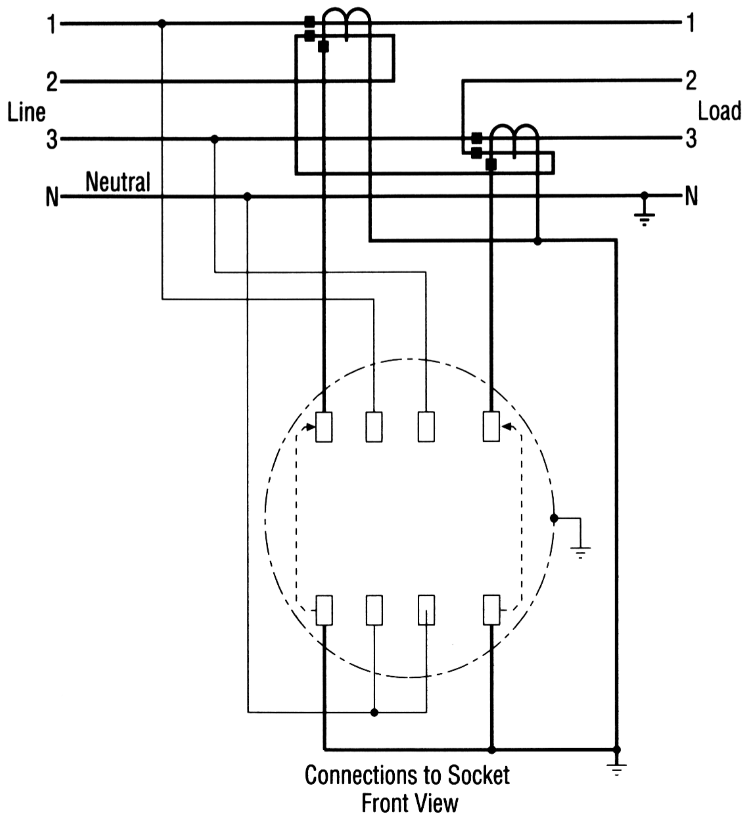
Three-Phase Three-Wire Delta With Two CTs



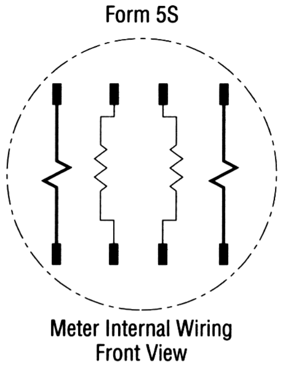
- Three wires
- Two voltage measurements with one side common to Line 2
- Current measurements on lines 1 & 3.

This satisfies Blondel's Theorem.

Blondel's Theorem



Three-Phase
Four-Wire Wye
With Two Equal-Ratio CTs



- Four wires
- Two voltage measurements to neutral
- Current measurements on lines 1 & 3. How about line 2?

This DOES NOT satisfy Blondel's Theorem.

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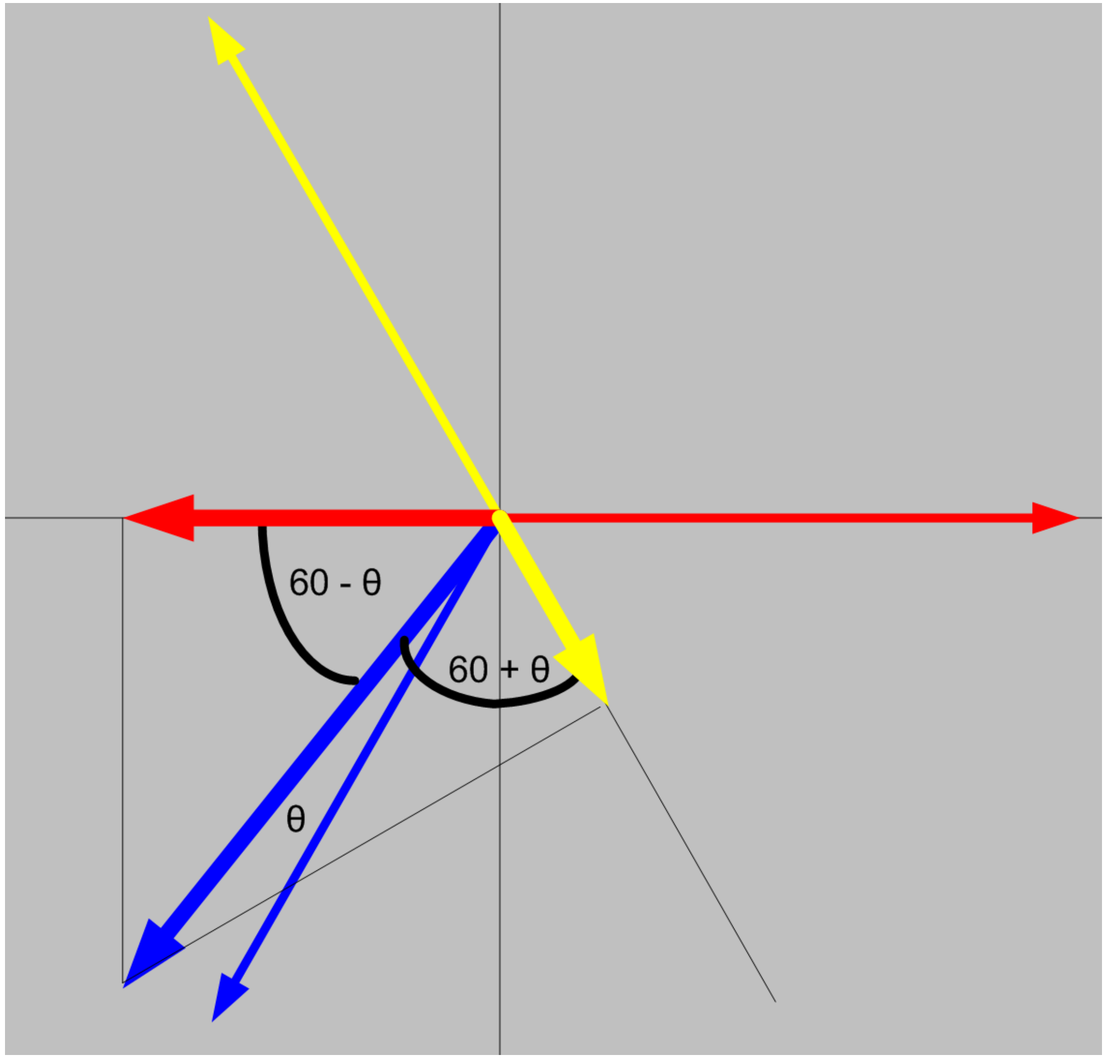
Blondel’s Theorem

- In the previous example:
  - What are the “ASSUMPTIONS”?
  - When do we get errors?

Blondel's Theorem



Blondel's Theorem

- Phase B power would be:
  - $P = V_b I_b \cos\theta$
- But we aren't measuring Vb
- What we are measuring is:
  - $I_b V_a \cos(60 - \theta) + I_b V_c \cos(60 + \theta)$
- $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
- So

Blondel's Theorem

- $Pb = IbVaCos(60 - \theta) + IbVcCos(60 + \theta)$
- Applying the trig identity
  - $IbVa(Cos(60)Cos(\theta) + Sin(60)Sin(\theta))$

    $IbVc (Cos(60)Cos(\theta) - Sin(60)Sin(\theta))$
  - $Ib(Va+Vc)0.5Cos(\theta) + Ib(Vc-Va)\ 0.866Sin(\theta)$
- Assuming
  - Assume Vb = Va = Vc
  - And, they are exactly 120° apart
- $Pb = Ib(2Vb)(0.5Cos\theta) = IbVbCos\theta$

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Blondel's Theorem

- If Va ≠ Vb ≠ Vc then the error is
- %Error =

  -Ib{(Va+Vc)/(2Vb) - (Va-Vc) 0.866Sin(θ)/(VbCos(θ))

How big is this in reality? If

Va=117, Vb=120, Vc=119, PF=1 then E=-1.67%

Va=117, Vb=116, Vc=119, PF=.866 then E=-1.67%

AC Theory – Power

- Power is defined as P = VI
- Since the voltage and current at every point in time for an AC signal is different, we have to distinguish between instantaneous power and average power. Generally when we say "power" we mean average power.
- Average power is only defined over an integer number of cycles.

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Harmonics

Curse of the Modern World

- Every thing discussed so far was based on “Linear” loads.
  - For linear loads the current is always a simple sine wave. Everything we have discussed is true.
- For nearly a century after AC power was in use ALL loads were linear.
- Today, many loads are NON-LINEAR.

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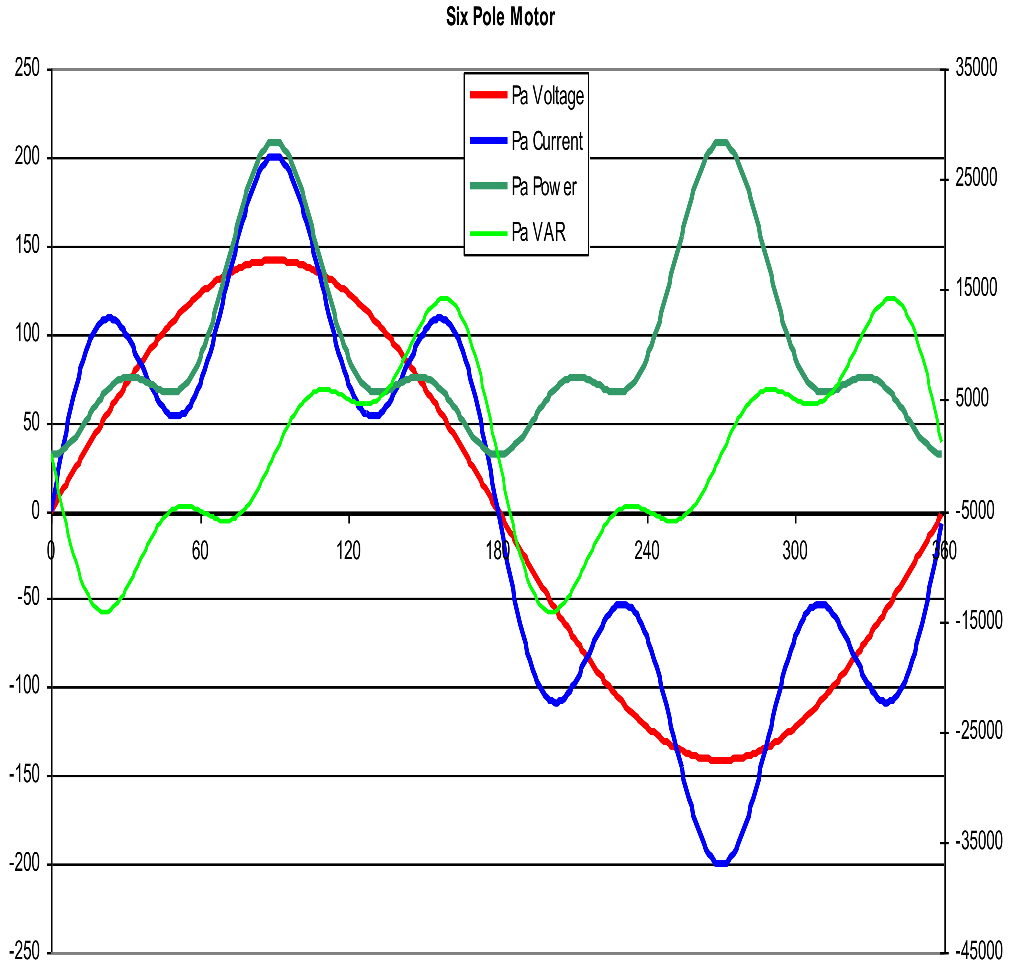
Harmonic Load Waveform

Eq.#	Quantity	Phase A
1	V(rms) (Direct Sum)	100
2	I(rms) (Direct Sum)	108
3	V(rms) (Fourier)	100
4	I(rms) (Fourier)	108
5	Pa = (∫ V(t)I(t)dt)	10000
6	Pb = ½∑V*n*In**cos**(θ)	10000
7	Q = ½∑VnIn**sin**(θ)	0.000
8	Sa = Sqrt(P^2 +Q^2)	10000
9	Sb = Vrms*Irms(DS)	10833
10	Sc = Vrms*Irms(F)	10833
13	PF = Pa/Sa	1.000
14	PF = Pb/Sb	0.923
15	PF = Pb/Sc	0.923



V = 100Sin(ωt)   I = 100Sin(ωt) + 42Sin(5 ωt)

Harmonic Load Waveform

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13	PF = Pa/Sa	1.000
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15	PF = Pb/Sc	0.923

- Important things to note:
  - Because the voltage is NOT distorted, the harmonic in the current does not contribute to active power.
  - It does contribute to the Apparent power.
  - Does the Power Triangle hold

$$S? = \sqrt{P^2 + Q^2}$$

  - There is considerable disagreement about the definition of various power quantities when harmonics are present.

$V = 100\text{Sin}(\omega t)$ $I = 100\text{Sin}(\omega t) + 42\text{Sin}(5\omega t)$

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3 Phase Power Measurement

- We have discussed how to measure and view power quantities (W, VARs, VA) in a single phase case.
- How do we combine them in a multi-phase system?
- Two common approaches:
  - Arithmetic
  - Vectorial

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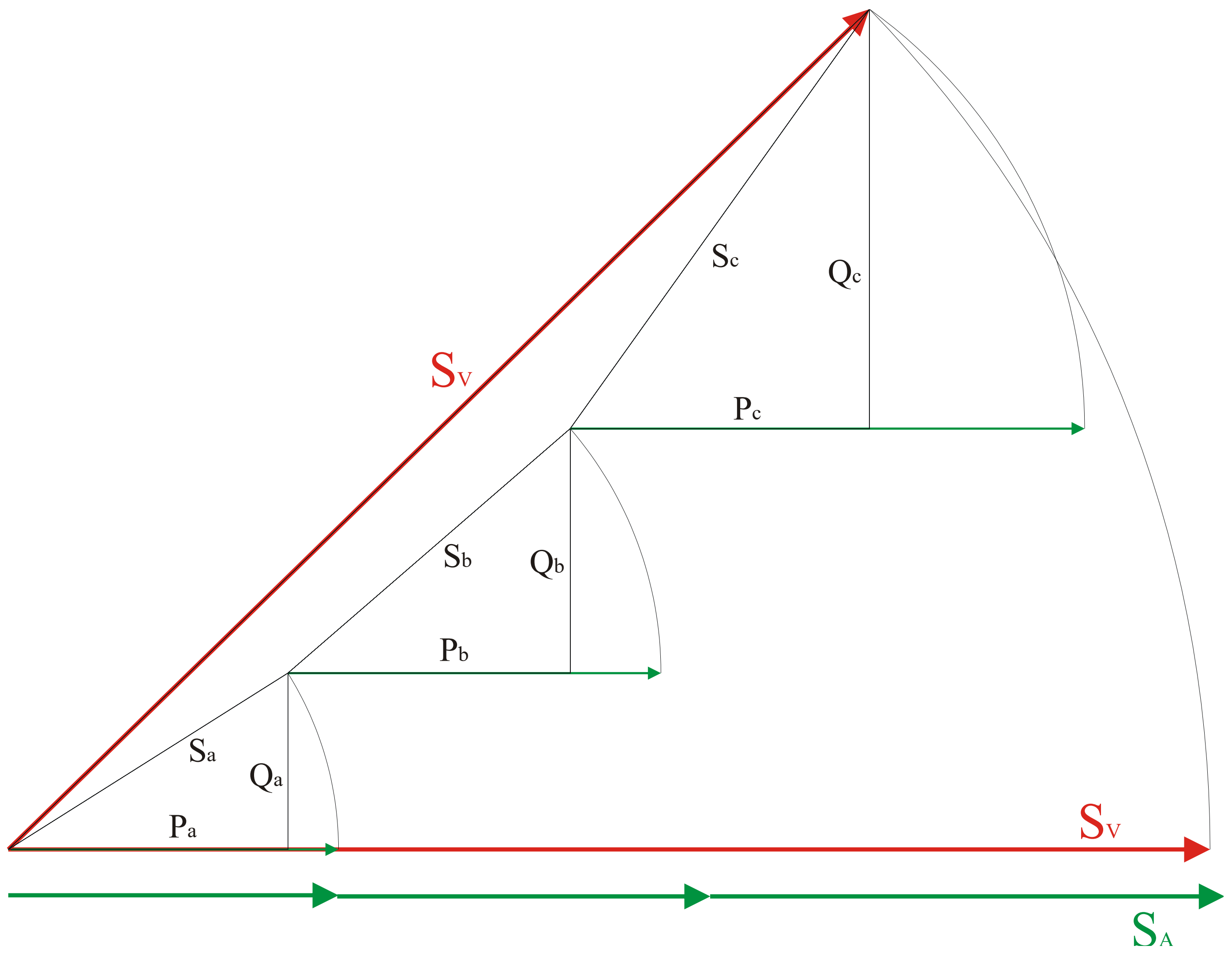
3 Phase Power Measurement

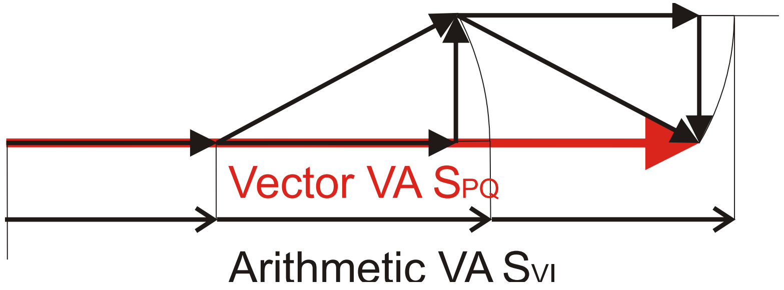


3 Phase Power Measurement

- VAR and VA calculations can lead to some strange results:
  - If we define

$$VA = \sqrt{(W_A + W_B + W_C)^2 + (Q_A + Q_B + Q_C)^2}$$

PH	W	Q	VA
A	100	0	100
B	120	55	132
C	120	-55	132
Arithmetic VA			364
Vector VA			340



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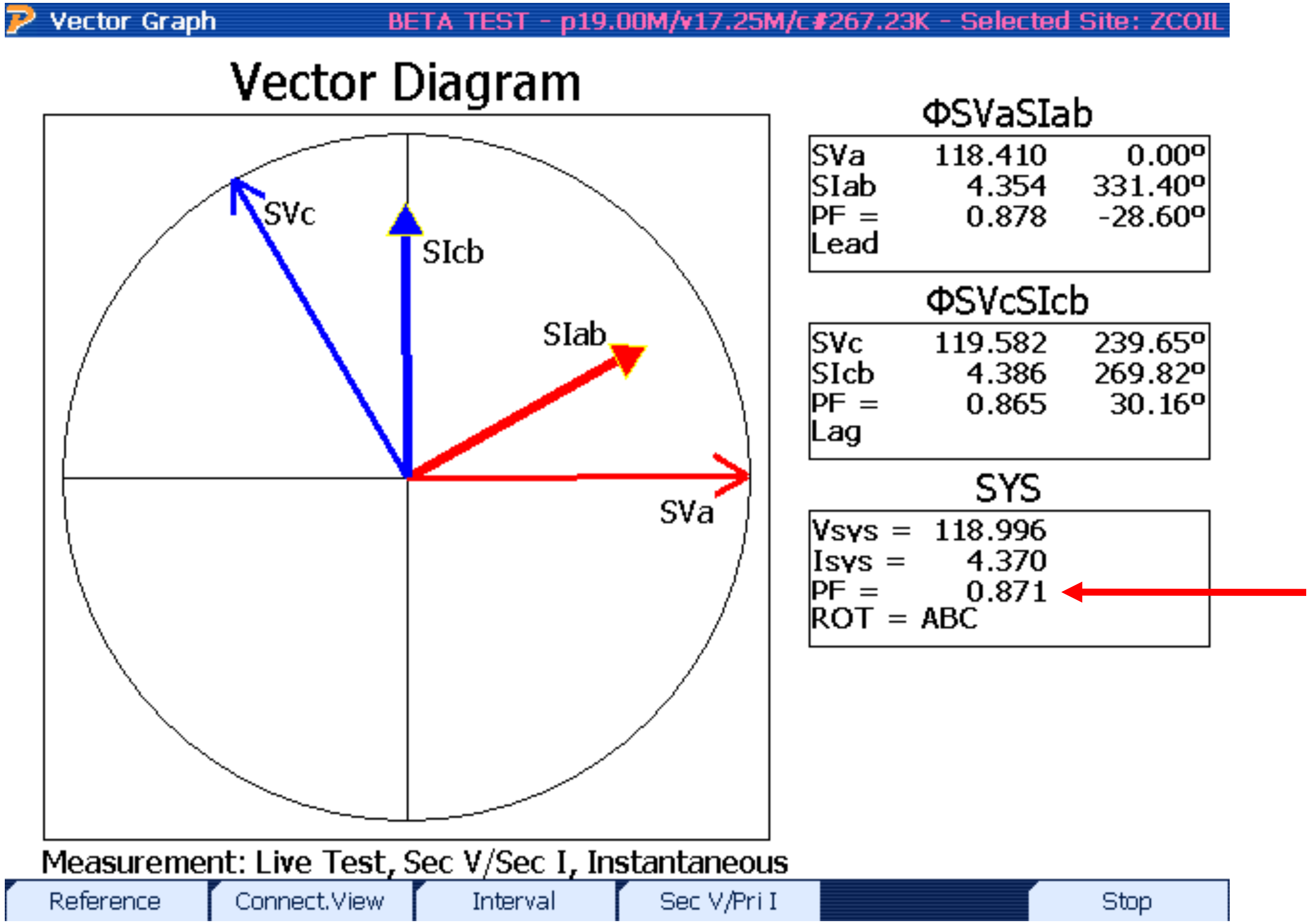
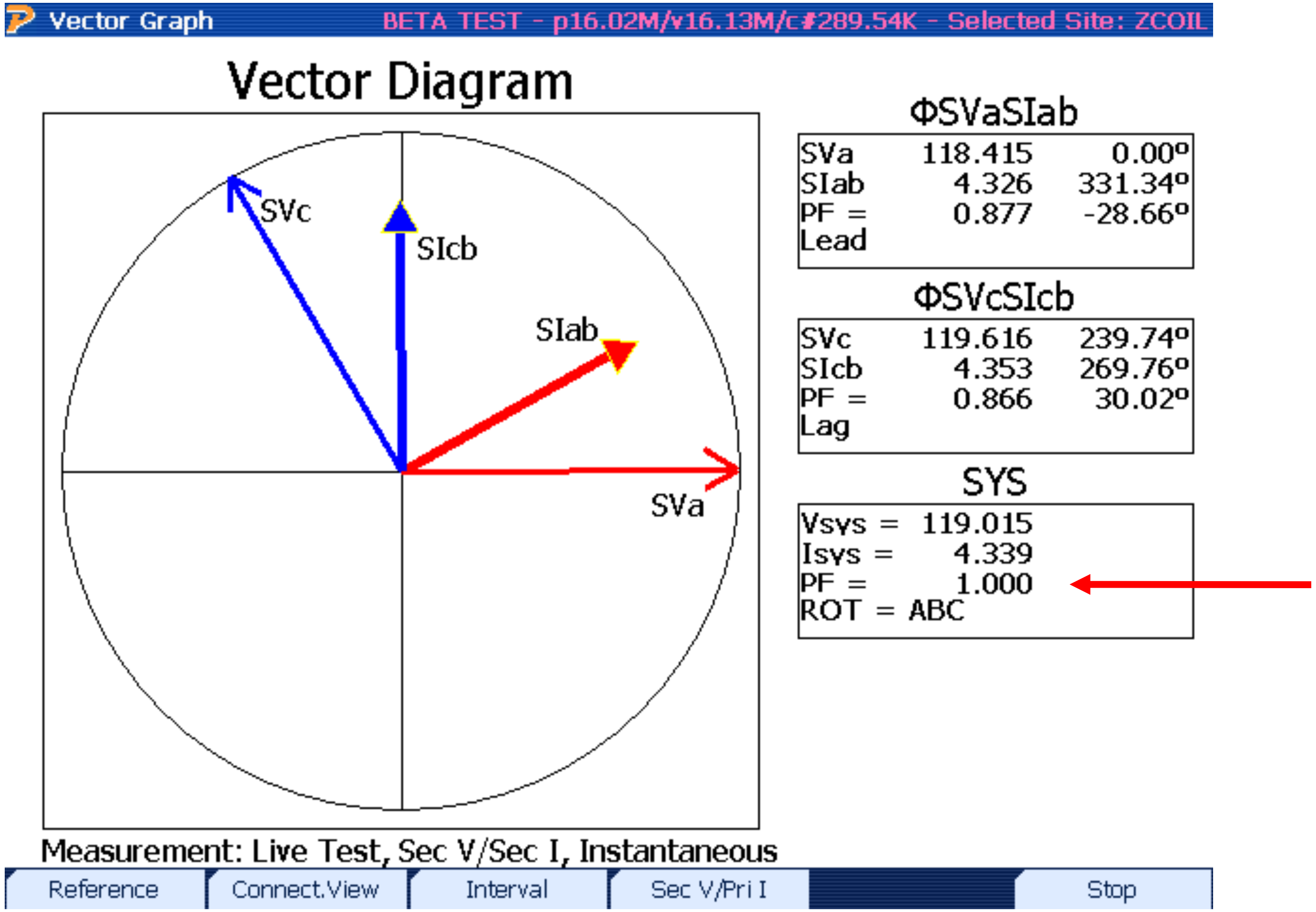
3 Phase Power Measurement



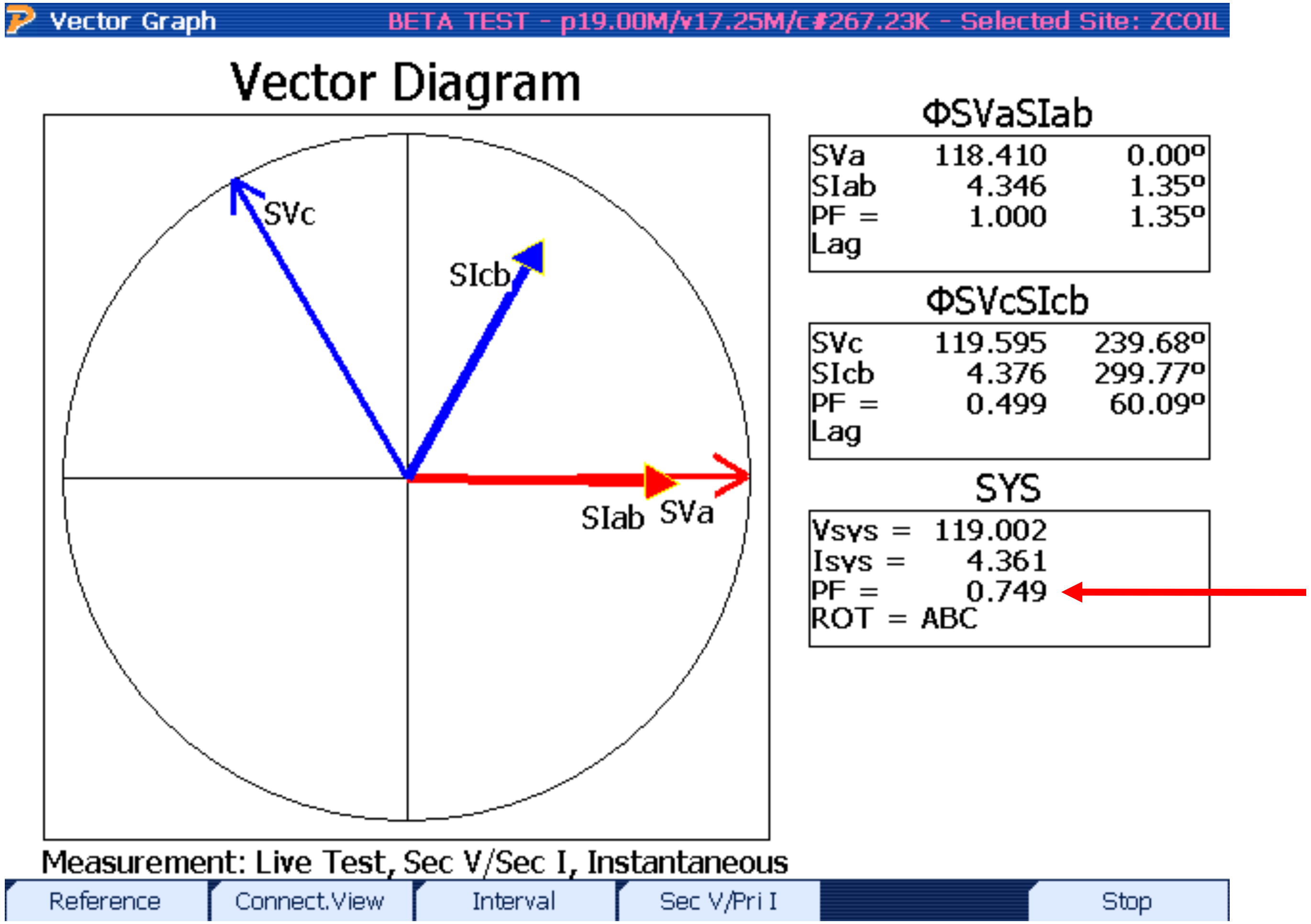
Arithmetic Calculation - Form 6 – 4 Wire Y Site
Voltages and Currents Aligned at 0°

3 Phase Power Measurement



Vector Calculation - Form 6 – 4 Wire Y Site
Voltages and Currents Aligned at 0°

3 Phase Power Measurement



Arithmetic Calculation - Form 6 – 4 Wire Y Site

Currents All shifted by 30°

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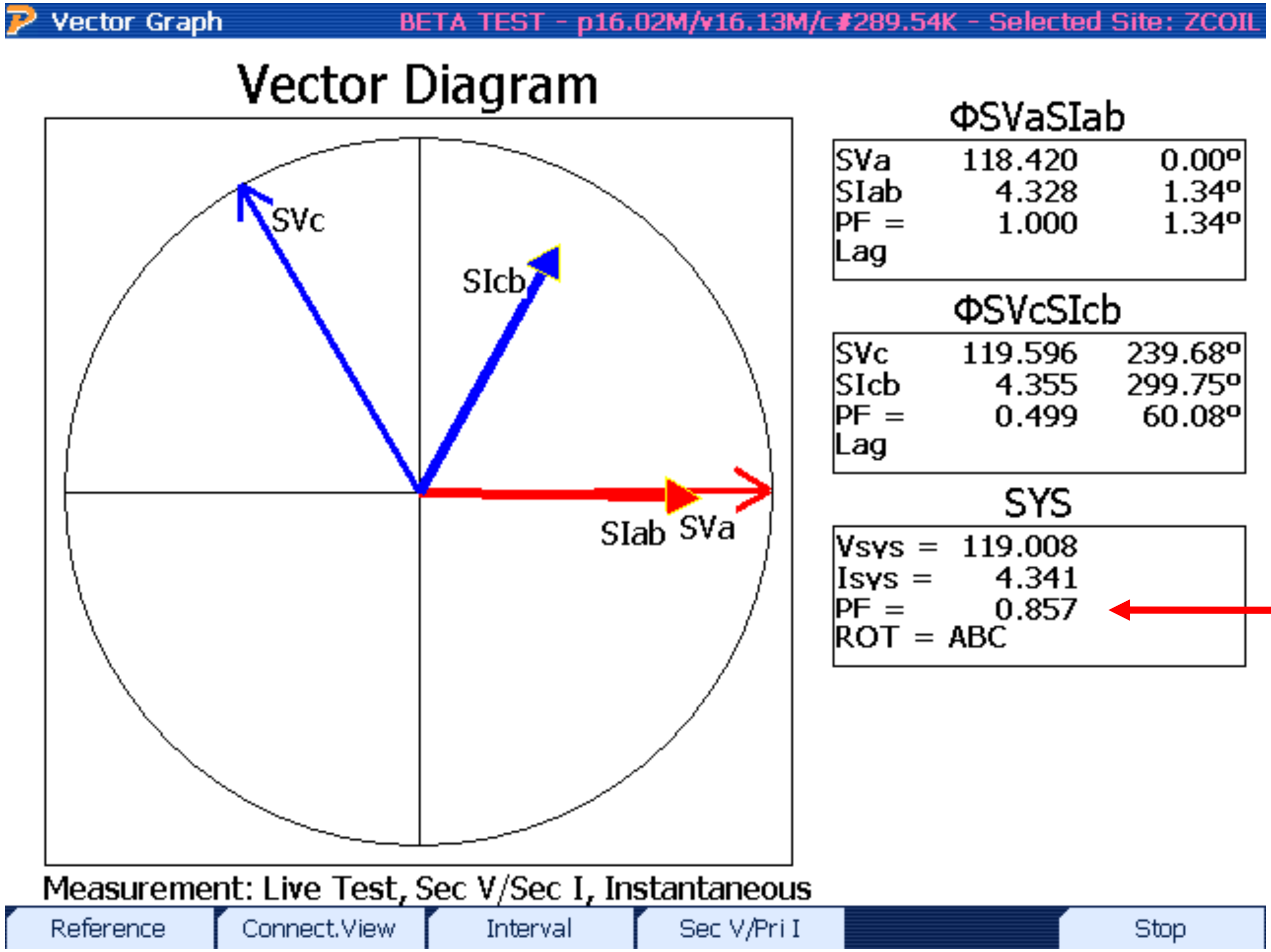
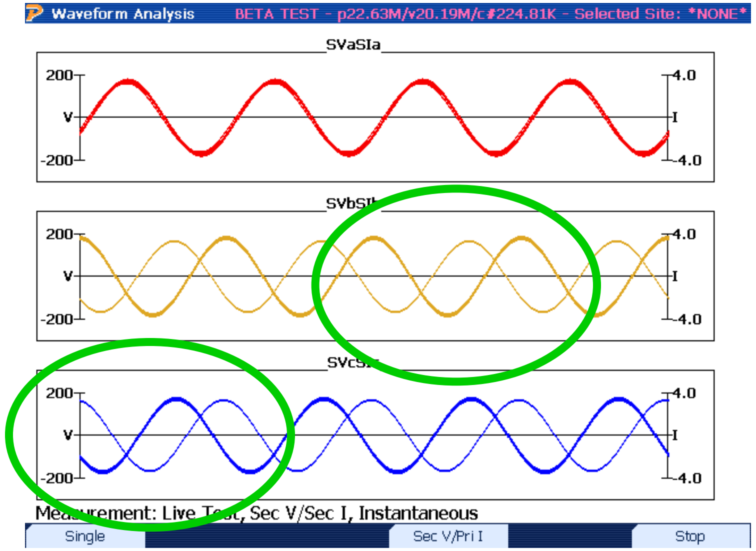
3 Phase Power Measurement



Vector Calculation - Form 6 – 4 Wire Y Site
Currents All shifted by 30°

Actual Field Test Case #1:
Lots of Clues!



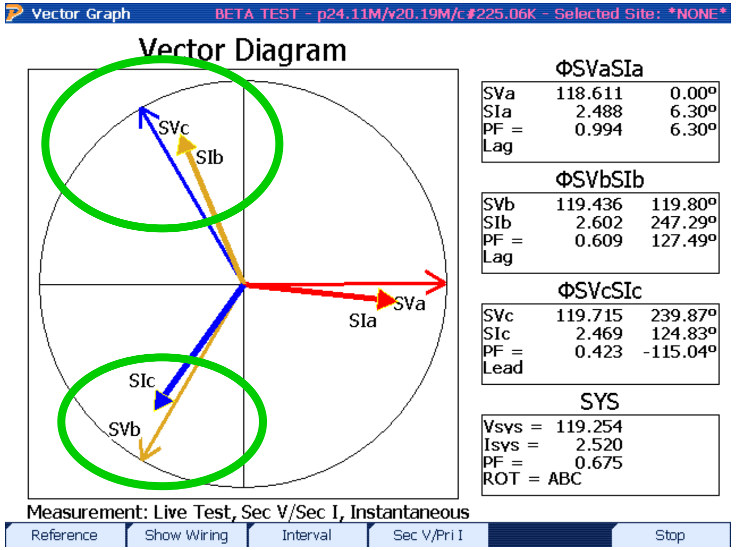
Power Meter — BETA TEST - p22.61M/v20.19M/c#224.81K - Selected Site: *NONE*

SYSTEM OVERALL SUMMARY

	ΦSVaSIa	ΦSVbSIb	ΦSVcSIc	SYSTEM
V(FDRMS)	118.6084	119.4340	119.7152	119.2526
V(Fund)	118.6021	119.4339	119.7151	119.2504
I(FDRMS)	2.487482	[illegible]	[illegible]	2.519800
A(Fund)	2.487467	[illegible]	[illegible]	2.519788
VΘ	0.0000°	119.8120°	239.8798°	
IΘ	6.3062°	247.3043°	124.8261°	
DPFΘ	6.3062°	127.4923°	-115.0537°	
PF(PF1a)	0.993949	0.608656	0.423469	0.675358
W(P1)	**293.2338**	**[illegible]**	**[illegible]**	**-21.1364**
VA(S1)	295.0188	[illegible]	[illegible]	-311.4321
VAR(Qt)	32.4005	246.6024	-267.8293	11.1736
THD V	1.033658%	0.138453%	0.142976%	0.438362%
THD I	0.338860%	0.232847%	0.328474%	0.300060%
FREQ	60.00016	60.00016	60.00015	60.00016

Measurement: Live Test, Sec V/Sec I, Instantaneous

Interval | Sec V/Pri I | Stop



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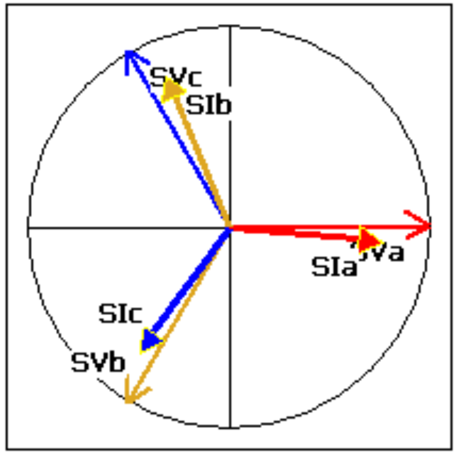
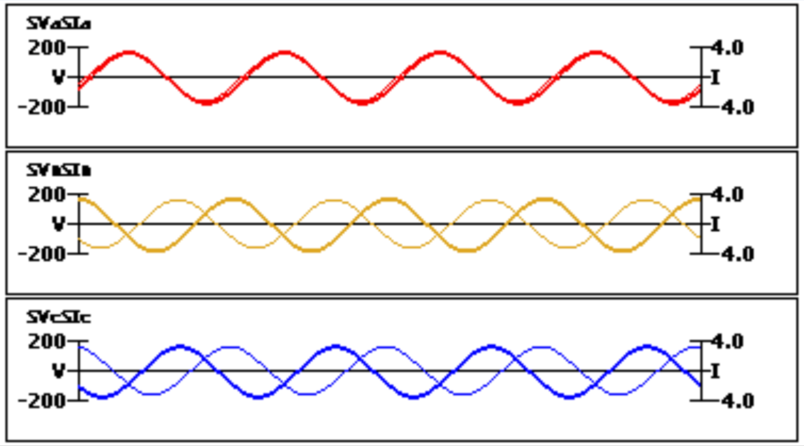
Actual Field Test Case #1:

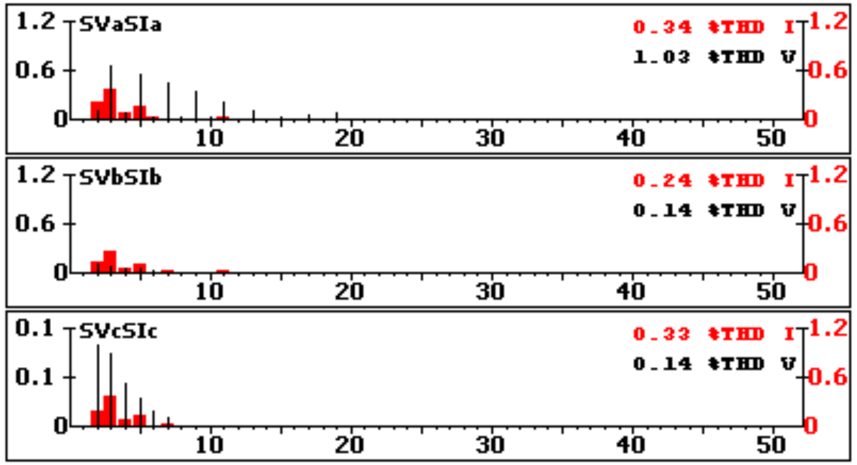
Lots of Clues!

Full Analysis   BETA TEST - p22.60M/v20.19M/c#224.81K - Selected Site: *NONE*





	ΦSVaSIa	ΦSVbSIb	ΦSVcSIc	SYSTEM
V(FDRMS)	118.62	119.43	119.71	119.25
V(Fund)	118.61	119.43	119.71	119.25
I(FDRMS)	2.4872	2.6023	2.4696	2.5197
A(Fund)	2.4872	2.6023	2.4696	2.5197
VΘ	0.00°	119.81°	239.88°	
IΘ	6.30°	247.30°	124.83°	
DPFΘ	6.30°	127.49°	-115.06°	
PF(PF1a)	0.9940	0.6086	0.4235	0.6754
W(P1)	**293.22**	**-189.16**	**-125.21**	**-21.15**
VA(S1)	295.00	-310.80	-295.65	-311.44
VAR(Qt)	32.39	246.60	-267.82	11.17
THD V	1.0255%	0.1385%	0.1430%	0.4356%
THD I	0.3445%	0.2373%	0.3332%	0.3050%
FREQ	60.000	60.000	60.000	60.000



Measurement: Last Test, Sec V/Sec I, Instantaneous

Restart

Phase B & C reversed!

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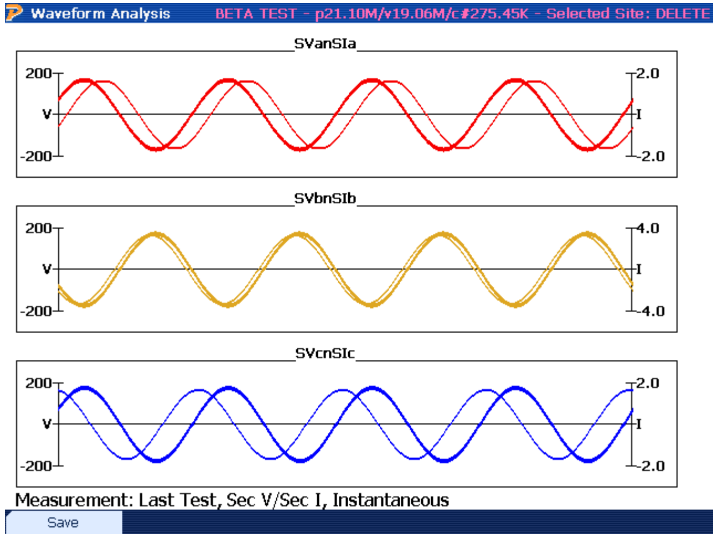
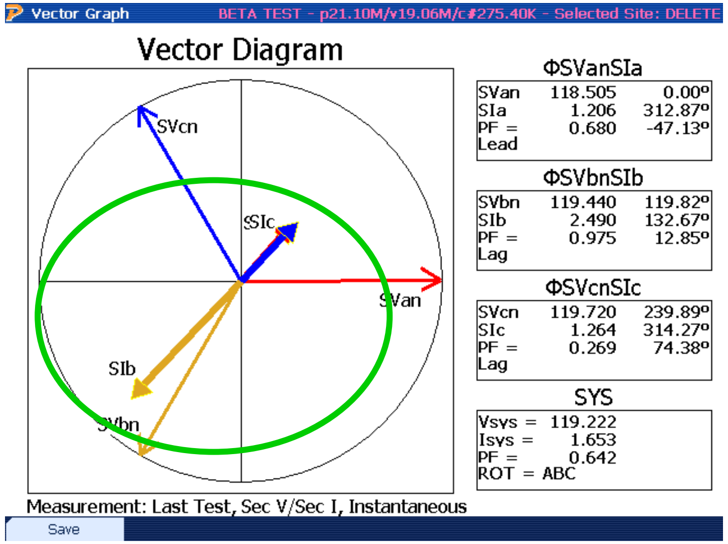
Actual Field Test Case #2:





Ib opposite to Ia & Ic

Power Meter — BETA TEST - p21.10M/v19.06M/c#275.40K - Selected Site: DELETE

SYSTEM OVERALL SUMMARY

	ΦSVanSIa	ΦSVbnSIb	ΦSVcnSIc	SYSTEM
V(FDRMS)	118.5131	119.4399	119.7201	119.2244
V(Fund)	118.5053	119.4398	119.7200	119.2217
I(FDRMS)	1.205595	2.489768	1.263508	1.652957
A(Fund)	1.205588	2.489754	1.263505	1.652949
VΘ	0.0000°	119.8231°	239.8886°	
IΘ	312.8747°	132.6724°	314.2653°	
DPFΘ	-47.12535°	12.84930°	74.37667°	
PF(PF1a)	0.680394	0.974958	0.269311	0.641554
W(P1)	**97.2070**	**289.9290**	**40.7378**	**427.8738**
VA(S1)	142.8686	297.3758	151.2668	591.5112
VAR(Qt)	-104.6986	66.1333	145.6779	107.1126
THD V	1.148540%	0.140896%	0.145333%	0.478256%
THD I	0.347228%	0.332820%	0.221998%	0.300682%
FREQ	59.99932	60.00014	60.00013	59.99986

Measurement: Last Test, Sec V/Sec I, Instantaneous

Save

Ib = Ia + Ic

Customer Load Test Results — BETA TEST - p21.14M/v19.00M/c#275.08K - Selected Site: DELETE

Customer Load Meter Test

Wh Test

% Registration 100.015

Test Info

Time(sec)	151.427
Time Left	0.000
Pulses Exp	9.9985
Pulses Act	10.0000
Meter PF	0.6416

Sys Info

Wh	17.9973
VAh	24.8777
VARh	4.4997
V	119.259
I	1.6524

Test Complete

Restart View Trend

Registration OK!

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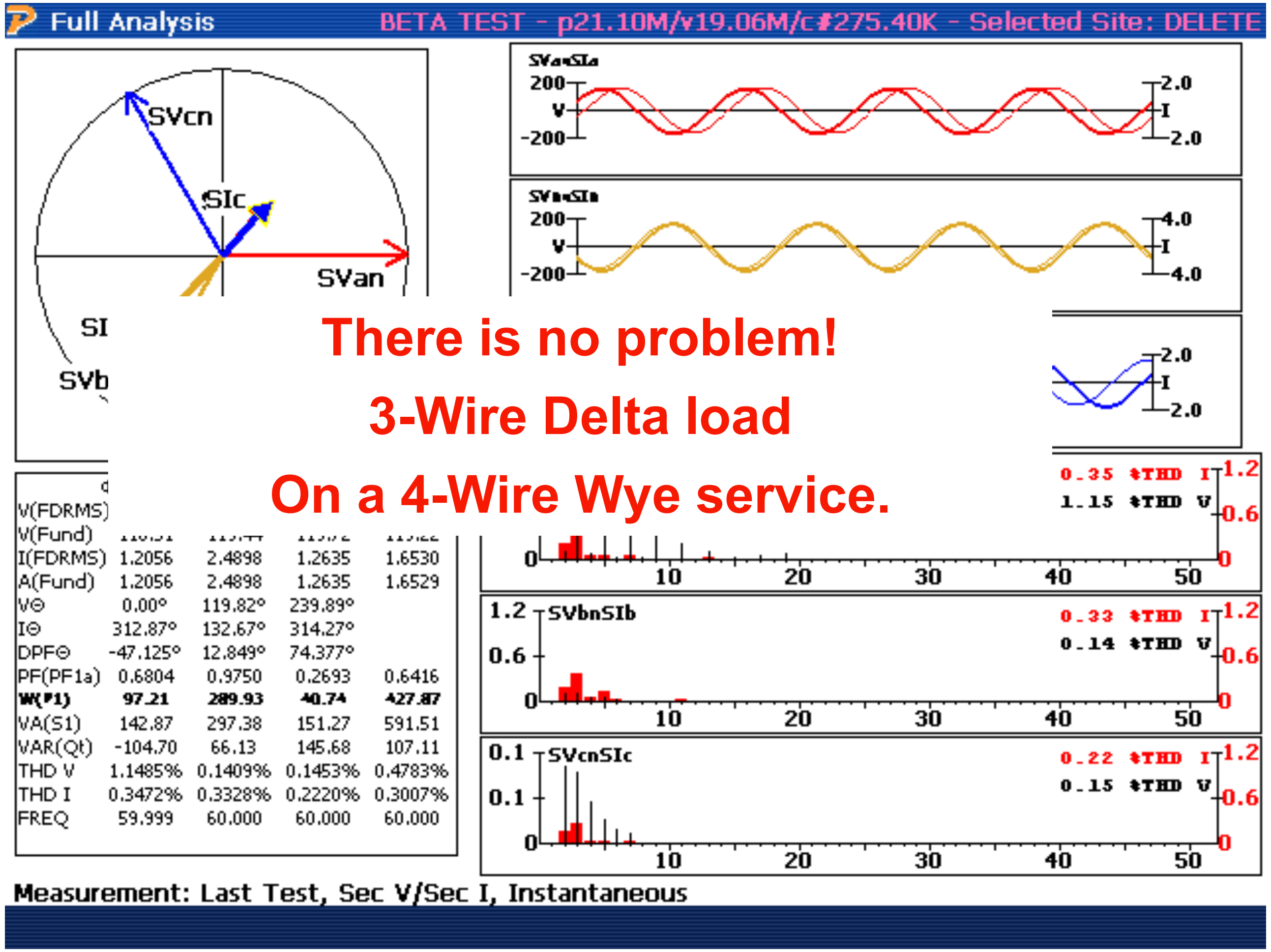
Actual Field Test Case #4:



There is no problem!

3-Wire Delta load

On a 4-Wire Wye service.

What is the problem?

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Questions? Comments?

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Thank you for your time!

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